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# Model calculations for the high-field peak of the fish-tail effect in the magnetostriction of type-II superconductors

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**Abstract**

We have aimed to describe the peak-effect observed in various magnetostriction experiments of type-II superconductors in the framework of critical-state models. A Gaussian term added to the exponential model and a Lorentzian term added to the Kim model for the field dependences of critical current density were employed in the calculations. The equations were solved numerically to obtain flux profiles inside a superconducting sample and thus to form the curves of magnetostriction versus applied field. The exponential model was also employed to reproduce the magnetostriction measurements performed on a single crystal 2H-$NbSe_2$ carried out by Eremenko et al. [Low Temp. Phys. 27 (2001) 305].


*Keywords:* Fish-tail effect; Magnetostriction; Exponential model

## 1. Introduction

The investigation of magnetostriction of type-II superconductors can offer very powerful method to study their pinning-related properties and to determine some superconducting parameters. The measurements performed on a $Bi_2Sr_2CaCu_2O_8$ single crystals revealed a relative change in sample length as large as $10^{-4}$ at 4.8 K. To explain this observation, Ikuta et al. [1] were the first to develop a model dealing with the magnetostriction of the high-temperature superconductors immersed in a magnetic field. The model proposed in ref. [1] was used by other authors for more realistic geometries [2–9]. The magnetostriction was also measured for various type-II superconductors [10–20].

In some magnetostriction measurements performed on type-II superconductors, an anomalous peak in $\Delta L/L$ versus $H_a$ curve was observed in fields slightly below the upper critical field $H_{c2}$ [17,18]. This unusual behavior of magnetostriction in superconductors was called fish-tail or peak-effect. There are three unique features attracting attention in the magnetostriction loops yielded in these experiments: (i) a peak in $\Delta L/L$ at high fields, (ii) a local minimum in peak onset and offset for increasing and decreasing field, respectively, and (iii) the asymmetry in downsweep and upsweep peaks of $\Delta L/L$ as magnetic field varies. It is evident that these observations cannot be explained by the commonly used model of Ikuta et al. [1]. The fish-tail effect has also been observed in various magnetization measurements of superconductors [22–25]. Some phenomenological models based on critical-state model were presented in literature to describe this effect in magnetization data [26–31].

There are a lot of striking similarities in irreversible behavior of the pinning induced magnetostriction and magnetization as seen in refs. [11,32]. The formula for magnetostriction (see Eq. (1)) is like a second-order magnetization. As a result, it might be expected that certain characteristic features of magnetization also have parallel features in magnetostriction [33,14]. Therefore, to treat the anomalities observed in magnetostriction experiments, a way similar to that for the magnetization [34,35] can be pursued.

The main objective of this work is to show that the experimental observations reported by Eremenko et al. [17] for the magnetostriction in 2H-$NbSe_2$ single crystals can be well reproduced by exploiting a Gaussian term added exponential model. For comparison, we also present the magnetostriction calculations using a Lorentzian term added Kim model for the field dependences of the critical current density.

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## 2. Basic framework of modeling

We consider a sample in the form of an infinite slab of width $2W$, where the external magnetic field $B_a$ is applied parallel to the slab. The relative variation in the sample width is given as follows [1]:

$$\frac{\Delta L}{L} = -\frac{1}{2c_0\mu_0 W}\int_0^W (B_a^2 - B^2(x))\,dx \qquad (1)$$

where $c_0$ is the stiffness constant, $\mu_0$ the permeability of the vacuum, $L = 2W$ the sample width, $H_a = B_a/\mu_0$ the externally applied magnetic field, and $B(x)$ is the magnetic flux density permeating the slab along the $z$-axis. The field distribution in a superconductor is defined by Maxwell's equation $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$, where $\vec{J}$ is current density and its value equals $J_c$ in the critical-state when Lorentz force density is equal to the pinning force density of the sample. The magnetic flux density profile $B(x)$ inside the slab sample is given by

$$\frac{dB}{dx} = \pm\mu_0 J_c(B) \qquad (2)$$

where the boundary condition is $B_a = B$ ($x = 0$). $\pm$ signs correspond to regions for $x$, where vortices have moved into or out of the sample. We assume the field dependence of the critical current density $J_c(B)$ as follows:

$$J_c(B) = J_{c0}\left(\frac{1}{1+(B/B_0)} + \frac{J}{((B/B_0)-B_p)^2 + B_W^2}\right) \qquad (3)$$

for a Lorentzian term added Kim–Anderson model [36,26] and

$$J_c(B) = J_{c0}(e^{-|B/B_0|} + J e^{-((B-B_p)^2/2B_W^2)}) \qquad (4)$$

for a Gaussian term added exponential models [37,27].Where the second terms on the right-hand side of Eqs. (3) and (4) is responsible for peak in the critical current $J_c$. Thus, in here positive phenomenological parameters $J$, $B_p$ and $B_W$ represent the relative amplitude, central position and width of the peak at half maximum, respectively. $B_0$ is phenomenological parameter and $J_{c0}$ the critical current density at zero magnetic field. All phenomenological parameters can also be assumed temperature dependent. In order to obtain the expression for the flux density profile in the region penetrated by the flux when the field is applied after zero-field cooling process, Eqs. (3) and (4) are substituted into Eq. (2) separately, and solved in the given boundary condition. We note that flux density profile formula for Kim-like dependence of critical current density, see Eq. (3), is also introduced in Eq. (4) in ref. [26]. Although it is not analytically possible to derive the $B$ profile expressions for the field dependence of the critical current density given by Eq. (4), the values of $B(x)$ in the specimen can be found numerically by means of one of the root finding methods such as bisection or Newton–Raphson. The calculated values of $B(x)$ are substituted into Eq. (2) and the integral of $\Delta L/L$ for the specific value of applied field is evaluated numerically employing Simpson or Romberg integration method. The full cycle of $\Delta L/L$ versus $H_a$ can be obtained pursuing the method presented by Ikuta et al. [32].

## 3. Results and discussion

Fig. 1a and b show the irreversible magnetostriction, $\Delta L/L$, as a function of the applied field $B_a$, using Eqs. (3) and (4), respectively, where selected parameters for $J$ are given in the legend. We note that the shape of the $\Delta L/L - B_a$ loops is strictly dependent on all the phenomenological parameters $B_0$, $J$, $B_p$ and $B_W$. The parameters $J$, $B_p$ and $B_W$ are more effective than $B_0$ to determine the form of the fish-tail peak in $\Delta L/L$ curves. These parameters $B_p$, $B_W$ and $J$ can be used to predict the central position of the peak in $\Delta L/L$, its width and magnitude, respectively. In calculations, the magnetostriction is normalized to $L_0 = B^{*2}/2c_0\mu_0 W$, the parameters $B_a$, $B_0$, $B_p$ and $B_W$ are normalized to $B^* = \mu_0 J_{c0} W$, first-penetration field, and $J$ is normalized to $J_{c0}$. We have used the same parameters in both models to make comparison one another.

Both the exponential-based and the Kim-based model are capable of describing the fish-tail peak in the $\Delta L/L - B_a$ loops quite well. However, as shown from Fig. 1a and b, there exist two remarkable differences between the pattern of the curves using two different models. First, the magnetostriction curves obtained using the exponential model have a more widespread

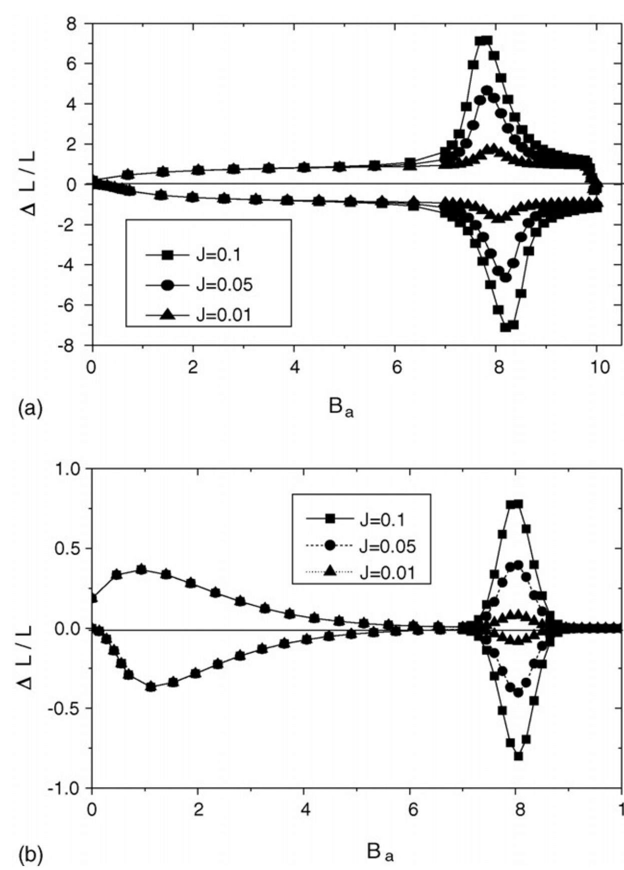


Fig. 1. Magnetostriction loops for cycles of applied field $B_a$ with different peak amplitude $J$ for (a) displays the calculations for a Lorentzian term added Kim model and (b) displays the calculations for a Gaussian term added exponential model. Both the magnetostriction and $B_a$ are normalized to $L_0 = B^{*2}/2c_0\mu_0 W$ and $B^* = \mu_0 J_{c0} W$, which is called the first-penetration field, respectively. The fitting parameters used in each calculation are as follows: $B_m = 10B^*$, $B_0 = 1B^*$, $B_p = 8B^*$ and $B_W = 0.3B^*$.