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General Temperature Computational Method of Linear Heat Conduction Multilayer Cylinder

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Abstract: According to the axisymmetric heat conduction of monolayer cylinder, a general method was deduced to calculate the axisymmetric temperature of linear heat conduction multilayer cylinder. Four types of boundary conditions were summarized and formulas for each type were derived. Then, a general calculating program was developed. Four temperature formulas could be expressed by a uniform equation, and the intermediate interface temperatures of axisymmetrical linear conduction multilayer cylinder satisfied tridiagonal linear and nonlinear systems of equations, which could be solved with the pursuit method and the Newton's method, respectively. With the calculating program, the temperature at any point of linear heat conduction multilayer cylinder could be obtained.

Key words: multilayer cylinder; axisymmetric heat conduction; temperature; linear heat conduction

The axisymmetric temperature distributions of single and double layer cylinder belong to a classic issue and have been reported[1-4]. But in practice, the heat conductivity of material linearly depends on temperature. The temperature computation of linear heat conduction multilayer cylinder is seldom covered yet. In metallurgical industry, there are several kinds of long cylindrical furnaces, such as blast furnaces and mixed iron cars, etc. Most of the cylindrical furnaces consist of several different refractory material layers and their heat conductivities linearly depend on temperature. Temperature distributions of these furnaces depend on the temperature analysis of heat transfer of linear conduction multilayer cylinder. Based on the principle of axisymmetric thermal conduction of monolayer cylinder, a general temperature computational method of linear heat conduction multilayer cylinder was deduced. The corresponding calculating program and an example were given. To induce equations conveniently, unit length of cylinder was considered.

1 Steady Temperature Distribution of Axisymmetric Linear Conduction Monolayer Cylinder

The temperature field in lateral section (Fig. 1) of axisymmetric cylinder only relates with radius r of point, $T=T(r)$. Under polar coordinate, its steady heat conduction equation

$$\frac{\partial}{\partial r}\left[k(T)r\frac{\partial T}{\partial r}\right]=0 \tag{1}$$

where, $k(T)$ is the linear dependency between material

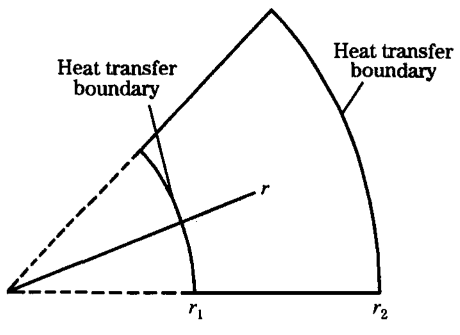

Fig. 1 Heat conduction of monolayer cylinder

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heat conductivity and temperature, and it can be given as

$$k(T)=k_0+aT \tag{2}$$

where, k_0 is the heat conductivity at room temperature; and a is the proportional coefficient.

Substituting Eqn. (2) into Eqn. (1), the following equation can be obtained

$$T(r)=\frac{\sqrt{2aA\ln r+2aB+k_0^2}}{a}-\frac{k_0}{a} \tag{3}$$

where A and B are coefficients determined by boundary conditions.

When material heat conductivity is constant, namely $a=0$, temperature distribution can be expressed as

$$T(r)=A\ln r+B \tag{4}$$

The heat flux Q is

$$Q=-Sk(T)\frac{\mathrm{d}T}{\mathrm{d}r} \tag{5}$$

where S is the area of lateral section in the heat transfer rate vector direction.

As $\bar{k}$ is the integral mean from temperature T_1 to T_2 by heat conductivity, Eqn. (5) can be changed into the following equation

$$Q=\frac{\bar{k}(T_1-T_2)}{\int_{r_1}^{r_2}\frac{\mathrm{d}r}{S}}=\frac{2\pi\bar{k}(T_1-T_2)}{\ln\frac{r_2}{r_1}} \tag{6}$$

where, T_1 is the inner surface temperature; and T_2 is the outer surface temperature.

Supposed that the inside and outside free surfaces keep constant temperature or convective heat transfer, the heat conduction of monolayer cylinder may has four combined boundary conditions: 1) fixed temperatures of both boundary sides, 2) fixed temperature at inside boundary and convective heat transfer at outside boundary, 3) convective heat transfer at inside boundary and fixed temperature at outside boundary, and 4) convective heat transfer at both boundary sides.

1) Fixed temperatures of both boundary sides

The thermal coefficients, A and B, can be expressed as

$$\begin{cases} A=-\dfrac{(aT_1+aT_2+2k_0)(T_1-T_2)}{2\ln\frac{r_2}{r_1}} \\ B=\dfrac{2T_1k_0\ln r_2+aT_1^2\ln r_2-2k_0T_2\ln r_1-aT_2^2\ln r_1}{2\ln\frac{r_2}{r_1}} \end{cases} \tag{7}$$

where, r_1 is the inner surface radius; and r_2 is outer surface radius.

From Eqn. (3) and Eqn. (7), the temperature at any point can be obtained.

2) Fixed temperature at inside boundary and convective heat transfer at outside boundary

According to the Newton's law of cooling, the heat flux can be expressed as

$$Q=2\pi rq=2\pi r_2h_w(T_2-T_w) \tag{8}$$

where, T_w is the outside fluid temperature; h_w is the outside heat transfer coefficient; and q is the density of heat flow rate.

The heat flux across the cylinder surface is constant and has no relation with radius. With Eqn. (2), Eqn. (6), and Eqn. (8), the temperature of outside boundary is

$$T_2=\frac{1}{a}\left[a^2T_1^2+2ak_0T_1+2ar_2h_wT_w\ln\frac{r_2}{r_1}+\left(k_0+r_2h_w\ln\frac{r_2}{r_1}\right)^2\right]^{1/2}-\frac{1}{a}\left|k_0+r_2h_w\ln\frac{r_2}{r_1}\right| \tag{9}$$

When the boundary temperatures are both constant, the temperature distribution can be obtained from Eqn. (3), Eqn. (7), and Eqn. (9).

3) Convective heat transfer at inside boundary and fixed temperature at outside boundary

As the same way, the temperature of inside boundary is

$$T_1=\frac{1}{a}\left[a^2T_2^2+2ak_0T_2+2ar_1h_nT_n\ln\frac{r_2}{r_1}+\left(k_0+r_1h_n\ln\frac{r_2}{r_1}\right)^2\right]^{1/2}-\frac{1}{a}\left|k_0+r_2h_n\ln\frac{r_2}{r_1}\right| \tag{10}$$

where, T_n is the inside fluid temperature; and h_n is the inside heat transfer coefficient.

With Eqn. (3), Eqn. (7), and Eqn. (10), the temperature distribution can also be obtained.

4) Convective heat transfer at boundaries of both sides

Because the heat flux across the cylinder surface is constant, the temperature of both boundary sides can be calculated. With Eqn. (6) and Eqn. (8), the temperature of inside boundary is

$$T_1=\frac{A_1-h_wr_2(B_1+C_1+D_1+E_1+F_1)^{1/2}}{a(h_n^2r_1^2-h_w^2r_2^2)} \tag{11}$$

where

$$A_1=h_w^2r_2^2\left(k_0+\ln\frac{r_2}{r_1}r_1h_n\right)+r_1h_n(k_0r_2h_w+ar_1h_nT_n+ar_2h_wT_w)$$

$$B_1=r_2^2h_w^2\left(k_0^2+a^2T_w^2+2k_0r_1h_n\ln\frac{r_2}{r_1}+2k_0aT_w\right)$$

$$C_1=2r_1r_2h_nh_w\left(k_0^2+aT_nh_wr_2\ln\frac{r_2}{r_1}+aT_nhk_0\right)$$

$$D_1=h_n^2r_1^2\left(\ln^2\frac{r_2}{r_1}h_w^2r_2^2+2\ln\frac{r_2}{r_1}r_2h_wk_0+k_0^2\right)$$

$$E_1=2ar_1r_2h_nh_wT_w\left(aT_n+k_0+r_1h_n\ln\frac{r_2}{r_1}\right)$$