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Splitting Rolling Simulated by Reproducing Kernel Particle Method

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Abstract: During splitting rolling simulation, re-meshing is necessary to prevent the effect of severe mesh distortion when the conventional finite element method is used. However, extreme deformation cannot be solved by the finite element method in splitting rolling. The reproducing kernel particle method can solve this problem because the continuum body is discretized by a set of nodes, and a finite element mesh is unnecessary, and there is no explicit limitation of mesh when the metal is split. To ensure stability in the large deformation elastoplastic analysis, the Lagrange material shape function was introduced. The transformation method was utilized to impose the essential boundary conditions. The splitting rolling method was simulated and the simulation results were in accordance with the experimental ones in the literature.

Key words: elasto-plasticity; large deformation; reproducing kernel particle method; splitting rolling

Symbol List

$C(x, x-s)$——Corrected function;
$d_{iJ}(t)$——Generalized displacement;
$\Delta \boldsymbol{d}$——Displacement increment;
f_I——Node value of $f(x)$ at node I;
$f^R(x)$——Reproducing function of $f(x)$;
$f^h(x)$——Reproducing kernel approximation of $f(x)$;
$\dot{f} \equiv \partial f/\partial t|_{[X]}$——Material derivative;
$f' \equiv \partial f/\partial x_i$——Spatial derivative;
$\hat{f}^{ext}$——External force vector;
$\hat{f}^{int}$——Internal force vector;
H^1——Sobolev space of degree one;
$\boldsymbol{I}$——Identity tensor;
$\hat{\boldsymbol{K}}$——Stiffness matrix;
$\hat{\boldsymbol{M}}$——Mass matrix;
NP——Whole number of nodes;
n_i——Surface outward normal in the current configuration;
s——Dummy variable of integration;
u_i^0——Initial displacement;
$\boldsymbol{u}(\boldsymbol{X}, t)$——Material displacement;
ΔV_I——Volume related with node I;
v_i^0——Initial velocity;
$w_h(\boldsymbol{x}-\boldsymbol{s})$——Kernel function;
$\phi_I(x)$——Shape function of RKPM;
σ_{ij}——Cauchy stress;
ρ——Density;
δ_{IJ}——Kronecker function;
Subscript
I, J, K——Node;
i, j, k, l——Spatial coordinate;
n——Time step;
v——Iteration number.

Metal splitting rolling technique is a novel metal forming method where the workpiece is split lengthwise into two or more sections using roller or other equipments during hot rolling. Extremely large deformation and metal split occur during splitting rolling. Numerical simulation with the finite element method (FEM) breaks down due to severe mesh distortion, and therefore, there is a need to re-mesh, but the projection field variables between meshes lead to a degradation in accuracy. And when the metal is split, the FEM fails owing to the existence of the mesh[1,2]. A novel meshless method has been developed to avoid these problems. The meshless method discretizes a continuum body by a finite number of

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nodes, and the displacement field is interpolated under those nodes without the aid of an explicit mesh. This characteristic simplifies model refinement procedures, and the use of smoother shape functions effectively handles large material distortion simulation. Recently, the meshless method is widely used to solve metal forming problems; for example, Chen J S et al[3,4] used the reproducing kernel particle method (RKPM) to simulate ring compression, and XIONG Shang-wu et al[5] simulated the rigid-plastic material's plane strain rolling process using RKPM. Bonet J et al[6] utilized the corrected smooth particle hydrodynamics (CSPH) to perform several two-dimensional simulations of basic metal forming, such as extrusion and forging. LI Chang-sheng et al[7] used the CSPH to solve the upsetting of the billet. Li G Y et al[8] utilized the element free Galerkin method (EFGM) to simulate the elasto-plastic material's plane strain extrusion process.

Although the application of a meshless method to simplify the metal-forming process has progressed to a certain extent, the process of complex metal forming, such as splitting rolling simulated by a meshless method, has not been reported due to the complex boundary condition and utmost deformation.

The large deformation elastoplastic RKPM is used to simulate the splitting rolling process in this study. The Lagrangre material shape function that deforms with the material was introduced for large deformation to ensure stability. A transformation method was used to satisfy the essential boundary condition[9]. Comparison of the computational results with experimental ones shows that RKPM is more competitive for simulation of metal forming.

1 Reproducing Kernel Particle Method

Liu W K et al[10] proposed RKPM based on an integral transformation with a modified kernel that exactly reproduces polynomials, i. e.,

$$f^R(\boldsymbol{x})=\int_\Omega C(\boldsymbol{x},\boldsymbol{x}-\boldsymbol{s})w_h(\boldsymbol{x}-\boldsymbol{s})f(\boldsymbol{s})\mathrm{d}s \tag{1}$$

The corrected function is defined as

$$C(\boldsymbol{x},\boldsymbol{x}-\boldsymbol{s})=\boldsymbol{p}^{\mathrm{T}}(0)\boldsymbol{M}^{-1}(\boldsymbol{x})\boldsymbol{p}(\boldsymbol{x}-\boldsymbol{s}) \tag{2}$$

$$\boldsymbol{p}^{\mathrm{T}}(\boldsymbol{x}-\boldsymbol{s})=[1,\boldsymbol{x}-\boldsymbol{s},\cdots,(\boldsymbol{x}-\boldsymbol{s})^N] \tag{3}$$

$$\boldsymbol{M}(\boldsymbol{x})=\int_\Omega \boldsymbol{p}(\boldsymbol{x}-\boldsymbol{s})\boldsymbol{p}^{\mathrm{T}}(\boldsymbol{x}-\boldsymbol{s})w_h(\boldsymbol{x}-\boldsymbol{s})\mathrm{d}s \tag{4}$$

Eqn. (1) can reproduce exactly an N-th order polynomial; therefore, this method satisfies the N-th consistency conditions. The discretization form of Eqn. (1) is

$$f^h(\boldsymbol{x})=\sum_{I=1}^{NP}C(\boldsymbol{x},\boldsymbol{x}-\boldsymbol{x}_I)w_h(\boldsymbol{x}-\boldsymbol{x}_I)\Delta V_I f_I=\sum_{I=1}^{NP}\phi_I(\boldsymbol{x})f_I \tag{5}$$

where

$$\phi_I(\boldsymbol{x})=C(\boldsymbol{x},\boldsymbol{x}-\boldsymbol{x}_I)w_I(\boldsymbol{x}-\boldsymbol{x}_I)\Delta V_I \tag{6}$$

2 Large Deformation Elastoplastic RKPM

2.1 Problem statement and variational equation

Let us consider a body which initially occupies a region Ω_X with boundary Γ_X and is deformed from its initial configuration to a deformed configuration Ω_x with deformed boundary Γ_x. The body is subjected to a body force b_i in Ω_x, surface traction h_t on the natural boundary $\Gamma_x^{h_i}$, and given displacement g_i on the essential boundary $\Gamma_x^{g_i}$. In the fixed Cartesian coordinate system, the particle positions in Ω_X are denoted by X, and those in Ω_x at time t by a mapping function $\boldsymbol{x}=\varphi(\boldsymbol{X},t)$. The model is given as follows:

$$\rho\ddot{u}_i=\sigma_{ij,j}+b_i \quad \text{in } \Omega_x \tag{7}$$

Boundary conditions

$$\sigma_{ij}n_j=h_i \quad \text{on } \Gamma_x^{h_i} \tag{8}$$

$$u_i=g_i \quad \text{on } \Gamma_x^{g_i} \tag{9}$$

Initial conditions

$$u_i(\boldsymbol{X},0)=u_i^0(\boldsymbol{X}) \tag{10}$$

$$\dot{u}_i(\boldsymbol{X},0)=v_i^0(\boldsymbol{X}) \tag{11}$$

The variational equation is formulated as follows: b_i, h_i, g_i, u_i^0, and v_i^0 are given, $u_i(\boldsymbol{X},t)\in H_g^1$ is found ($H_g^1=\{\boldsymbol{u}:\boldsymbol{u}\in H^1$, for $\Gamma_x^{g_i}$ $u_i=g_i\}$), such that for all $\delta u_i\in H_0^1(H_0^1=\{\boldsymbol{w}:\boldsymbol{w}\in H_0^1$, for $\Gamma_x^{g_i}$ $w_i=0\})$, the following equation is found satisfactory.

$$\delta\Pi=\int_{\Omega_x}\delta u_i\rho\ddot{u}_i\mathrm{d}\Omega+\int_{\Omega_x}\delta u_{i,j}\sigma_{ij}\mathrm{d}\Omega-\int_{\Omega_x}\delta u_i b_i\mathrm{d}\Omega-\int_{\Gamma_x^{h_i}}\delta u_i h_i\mathrm{d}\Gamma=0 \tag{12}$$

For the non-linear problem, the linearized Eqn. (12) can be written in the following form:

$$\delta\Pi=\int_{\Omega_x}\delta u_i\rho\Delta\ddot{u}_i\mathrm{d}\Omega+\int_{\Omega_x}\delta u_{i,j}(\boldsymbol{D}_{ijkl}+\boldsymbol{T}_{ijkl})\Delta u_{k,l}\mathrm{d}\Omega-\int_{\Omega_x}\delta u_i\Delta b_i\mathrm{d}\Omega-\int_{\Gamma_x^{h_i}}\delta u_i\Delta h_i\mathrm{d}\Gamma=0 \tag{13}$$

2.2 Lagrangian material shape function

In RKPM computation, the support of the kernel function must cover enough particles for the method to be stable[6]. In this study, the material kernel function that deforms with the material is used. The support of the material kernel function