Available online at www.sciencedirect.com

ScienceDirect

Procedia Materials Science 3 (2014) 378 – 384

Procedia Materials Science

www.elsevier.com/locate/procedia

20th European Conference on Fracture (ECF20)

Improved numerical determination of the effective stress within the framework of local approach cleavage models

Volker Hardenacke[a,]*, Jörg Hohe[a]

[a]*Fraunhofer IWM, Wöhlerstr. 11, 79108 Freiburg, Germany*

Abstract

For the determination of the Weibull stress or analogous effective stress measures in the assessment of the cleavage failure probability based on finite element results, robust and numerically stable procedures are necessary. For the quantitative assessment of the numerical errors in the approximate integration procedures for the determination of the Weibull stress or corresponding stress measures, an appropriate error indicator in analogy to adaptive finite element meshing methods was derived. The mathematical considerations reveal that an evaluation of the cleavage probability by means on the classical Beremin (1983) model using the standard one-point Gauss integration scheme may result in distinct numerical errors. Therefore, an enhanced numerical integration formula was derived and implemented. This formula facilitates a numerical evaluation of the integrals with improved accuracy.

Selection and peer-review under responsibility of the Norwegian University of Science and Technology (NTNU), Department of Structural Engineering

Keywords: Effective stress measure; numerical integration; error indicator; improved integration formula

1. Introduction

The process of cleavage fracture initiation is investigated since some decades. It is widely accepted that cleavage fracture is triggered by the instability of micro cracks. The oldest and most important model for cleavage fracture assessment is the model proposed by Beremin (1983). Within the framework of this model, it is assumed that the

* Corresponding author. Tel.: +49-761-5142-408; fax: +49-761-5142-510.
E-mail address: volker.hardenacke@iwm.fraunhofer.de

Selection and peer-review under responsibility of the Norwegian University of Science and Technology (NTNU), Department of Structural Engineering
doi:10.1016/j.mspro.2014.06.064

size distribution of the micro cracks follows a power law. The assumption that the instability of a micro crack (size l) can be described by the stress criterion $\sigma_{critical}=k^2/l^2$ (material parameter k) yields the accumulated probability $p(\sigma)=(\sigma/\sigma_u)^m$ for the failure of a control volume V_0. The parameters m and σ_u (contains k) are material parameters. The weakest-link approach proposed by Mudry (1987) postulates that the failure of the control volume V_0 causes the instable failure of the entire structure. Thereby, the accumulated probability of failure for the considered structure can be written as

$$P_f(\sigma_W) = 1 - exp\left(-\left(\frac{\sigma_W}{\sigma_u}\right)^m\right) \quad with \quad \sigma_W = \sqrt[m]{\frac{1}{V_0}\int_{V_{cl}} \sigma_I{}^m \, dV} \tag{1}$$

The Weibull stress σ_w is determined by the integration of the maximum values of the principal stress σ_I occurred in the volumes V_i until the considered point in time over the volume V_{cl} of the cleavage fracture process zone.

The local approach cleavage fracture models available in literature are usually based on the determination of an effective stress measure like the Weibull stress or a similar quantity, which is calculated by a generalized weighted averaging of the maximum principal stress over the plastic zone (interpreted as the cleavage fracture process zone). The numerical determination of the effective stress measure relevant for cleavage fracture initiation can be problematic with respect to the numerical stability. The reason for this fact is that a high power of the stress must be used for the calculation of the effective stress measure. Hence, even small errors at the determination of the maximum principal stress can cause significant errors during the calculation of the effective stress measure. In addition, approximation errors arise if the calculation of the Weibull stress is performed via a numerical integration procedure.

2. Error indicators for the Finite Element stress field and the effective stress measure (Weibull stress)

The accuracy of the stresses determined within the framework of Finite-Element analyses is to a great extent dependent on the chosen element size. For reasons of numerical efficiency the element size cannot be reduced arbitrarily. Hence, in the past years different error indicators for the stresses calculated using the Finite-Element method were derived, see Zienkiewicz (2006). These error indicators enable an estimation of the existing stress error for a given FE-mesh. The accuracy of the error estimation also depends on the mesh refinement to a certain extent. Within the framework of the present study, the simple error indicator proposed by Zienkiewicz (1987) is utilized. For the derivation of this error indicator we consider the real stress $\boldsymbol{\sigma}$ and its FE-approximation $\hat{\boldsymbol{\sigma}}$. The definition of an error indicator requires an improved approximation of the real stress. Such an improved approximation can be achieved by a nodal averaging or rather a projection procedure. In this context, it is assumed that a smoothed approximated stress $\boldsymbol{\sigma}^*$ can be interpolated by the same shape function as the displacements:

$$\boldsymbol{\sigma}^* = \mathbf{N}\bar{\boldsymbol{\sigma}}^* \quad \text{and} \quad \int_\Omega \mathbf{N}^T(\boldsymbol{\sigma}^* - \hat{\boldsymbol{\sigma}})\mathrm{d}\Omega = 0 \tag{2}$$

Furthermore holds (shape function **N**, matrix operator **S** and elasticity matrix **D**)

$$\bar{\boldsymbol{\sigma}}^* = \mathbf{A}^{-1}\int_\Omega \mathbf{N}^T\mathbf{DSN}\,\mathrm{d}\Omega\bar{\boldsymbol{u}} \quad \text{with} \quad \mathbf{A} = \int_\Omega \mathbf{N}^T\mathbf{N}\mathrm{d}\Omega \tag{3}$$

The smoothed approximated stress $\boldsymbol{\sigma}^*$ is an improved approximation of the real stress. This yields an error indicator for each element: $\boldsymbol{e_\sigma} \approx \boldsymbol{\sigma}^* - \hat{\boldsymbol{\sigma}}$.

Utilizing the stress error $\Delta\sigma_I$ known from $\boldsymbol{e_\sigma}$ for each element, it is possible to determine the Weibull stress error resulting from error propagation during the calculation of the effective stress measure. Considering a single finite element and assuming small stress errors $\Delta\sigma_I$, a Taylor series expansion of the integration formula yields

$$\Delta I_{error-propagation} \approx \sum_{i=1}^{n}\frac{\partial I_{el}}{\partial\sigma_I^i}\Delta\sigma_I^i \quad with \quad I_{el} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\sigma_I^i}{\sigma_u}\right)^m \tag{4}$$