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# Analogies in fracture mechanics of concrete, rock and ice

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**Abstract**

Both in architecture and arts, the golden ratio has been taken into consideration most exclusively for its geometrical properties. Specifically, among all the proportions, the golden ratio can inspire beauty and aesthetic pleasure. Indeed, it has driven, in an implicit or explicit manner, the construction of buildings for centuries. Nevertheless, as discussed in the present paper, also fracture mechanisms in brittle and quasi-brittle materials call the golden ratio into play. This is the case of fracture energy and fracture toughness, in which the irrational number 1.61803 recurs when the geometrical dimensions vary. This aspect is confirmed by the results of different experimental campaigns performed on concrete and rock beams and ice sheets. In other words, it can be argued that the centrality of the golden ratio for quasi-brittle structures has profound physical meanings, as it can bring together the aesthetic of nature and architecture, and the equilibrium of stress flow in solid bodies.


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## 1. Introduction

As is well known (Rilem TC QFS 2004), size effect represents an important physical character of cracked members made of plain or lightly reinforced concrete. It affects mechanical properties (i.e., tensile strength, shear strength, fracture energy, etc.) because of the non-homogeneous character of the material and the occurrence of fractures.

A wide literature on size effect has considered the decrease of material strength, and the increase of fracture energy, with structural size. Weibull (1939) defined size effect as the result of statistical distribution of defects

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inside the material, whereas Bazant (1984) adopted a deterministic approach based on the energy release rate. More recently, Carpinteri et al. (1995) combined the statistical and energetic approaches within the framework of fractal geometry, which permitted to take into account the intimate nature of energy dissipation during fracture propagation. As a result, the following Multifractal Scaling Law (MFSL) can be assumed for the fracture energy (Carpinteri and Chiaia 1996):

$$\frac{G_F}{G_F^{\infty}} = \left(1 + \frac{l_{ch}}{D}\right)^{-1/2} \tag{1}$$

where $G_F$ = nominal fracture energy; $D$ = external characteristic structural size; $G_F^{\infty}$ = nominal asymptotic fracture energy ($D \rightarrow \infty$); $l_{ch}$ = internal microstructural length.

According to Eq.(1), fracture energy, and consequently fracture toughness, increase with the dimension $D$ of the specimen, up to the asymptotic value $G_F^{\infty}$. Several experimental analyses, performed on concrete, rock and ice, seem to confirm the reliability of such law. Although concrete is a man-made material while ice and rock are natural materials, they show grain size depending behaviour and present a structurally similar brittle response. However, the properties of concrete, rock and ice show, at different scales, show the typical peculiarities of a fractal, characterized by scale invariant fractal porosity (Carbone et al. 2010, Carpinteri and Chiaia 1996).

Nevertheless, to apply the MFSL in brittle and quasi-brittle materials, the parameters $G^{\infty}_{\mathrm{F}}$ and $l_{\mathrm{ch}}$, which cannot be measured through direct specific tests, have to be computed using a best-fitting algorithm. Obviously, the regression procedure is more statistically reliable if the experimental measures of $G_{\mathrm{F}}$ are performed on a wide size range. In other words, a large number of tests is required to use correctly the MFSL (Carpinteri et al. 1995), as well as the Bazant's (1984) size effect law.

A new size-effect model is therefore introduced in the present paper with the aim of predicting the fracture energy of concrete, and the fracture toughness of rock and ice, starting from only a few tests performed on a single size-scale.

## 2. The size effect law

If the magnitude of a physical property of a body increases with the geometrical dimensions of the body, a size effect law having the form of a power function (Rilem TC QFS 2004) can be introduced:

$$\frac{s_r}{s_r^0} = \left(\frac{D}{D_0}\right)^{\beta} \tag{2}$$

where $D_0$ = reference dimension; $D$ = dimension at a generic scale; $s^0_{\mathrm{r}}$ = magnitude of the physical property at reference dimensions; $s_{\mathrm{r}}$ = magnitude of the physical property at a generic scale; and $\beta$ = exponent that has to be defined by fitting experimental and/or numerical data. In some circumstances, when $D = n\, D_0$, and $n$ is integer, $s_{\mathrm{r}}$ increase of a factor $\phi$ with respect to $s^0_{\mathrm{r}}$ (i.e., $s_{\mathrm{r}} / s^0_{\mathrm{r}} = \phi = 1.61803$).

In mathematics, $\phi$ is generally known as the golden ratio, or the divine proportion. A first definition of the golden ratio was proposed by Euclid (300 b.C.). In particular, in the VI book of the Elements, the third definition states: "*A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less*".

In other words, in the line segment depicted in Fig. 1, it is possible to localize a point where the ratio of the whole line (A) to the large segment (B) is the same as the ratio of the large segment (B) to the small one (C). Only when this ratio is equal to the golden ratio (i.e., the irrational number $\phi$), can such a division be possible.

In physics, this number, intimately interconnected with the Fibonacci sequence (1, 2, 3, 5, 8, 13…), controls growth in several natural patterns. In fact, the limit of the ratios of two successive terms of the series tends to the golden ratio.