Available online at www.sciencedirect.com
ScienceDirect
Trans. Nonferrous Met. Soc. China 24(2014) 3265−3273
Transactions of Nonferrous Metals Society of China
www.tnmsc.cn

# Accuracy analysis of plane-strain bulge test for determining mechanical properties of thin films

Lin YANG[1,2], Shi-guo LONG[1,2], Zeng-sheng MA[1,2], Zi-han WANG[1,2]

1. Key Laboratory of Low Dimensional Materials & Application Technology of Ministry of Education, Xiangtan University, Xiangtan 411105, China;
2. School of Materials Science and Engineering, Xiangtan University, Xiangtan 411105, China

Received 8 October 2013; accepted 27 January 2014

**Abstract:** The effect of a variety of geometrics, initial conditions and material properties on the deformation behavior of thin films in the plane-strain bulge test was systematically scrutinized by performing the finite element analysis, and then the accuracy of the plane-strain bulge test in determining the mechanical properties of thin films in terms of our finite element results was analyzed. The results indicate that although the determination of the plane-strain modulus in the light of the plane-strain bulge equation is fairly accurate, the calculation of the residual stress is not satisfied as expected, especially for low residual stress. Finally, an approach is proposed for analyzing bulge test data, which will improve the accuracy and reliability of this bulge test technique.
**Key words:** thin films; mechanical properties; bulge test; accuracy; finite element analysis

## 1 Introduction

Thin films have been widely used in many important applications such as integrated circuits and microelectromechanical systems [1,2]. To improve the reliability, lifespan and stability of thin films and make full use of these materials, an insight into the mechanical properties of thin films is becoming extremely important. Unfortunately, the deformation behavior and dimension of thin film materials are generally different from those of the corresponding bulk materials, hence specialized mechanical testing techniques have had to be developed [3,4]. There are several specialized techniques for measuring the mechanical properties of thin films including nanoindentation tests [5,6], uniaxial tensile tests [7,8], beam bending tests [9,10] and bulge tests [11,12]. Despite the availability of these techniques to measure the mechanical properties of thin films, specimen preparation, the test setup, and even data analysis are still challenges in the field [13].

One of the most promising techniques for characterizing the mechanical behavior of thin films is the bulge test. Since the bulge test was first introduced by BEAMS [14] in 1959, it has been used to measure the mechanical properties of thin films. In this technique, the deflection of a suspended film is measured as a function of applied pressure. By measuring the applied pressure and the resulting deflection, the elastic modulus, residual stress, Poisson ratio and other important parameters such as strength and fracture toughness can be determined by this method [15−18]. Traditionally, the test has suffered from a number of problems related to sample processing and handling ever since it was born; however, with the recent rapid development of Si micromachining technology, these problems are largely solved [3,4,11,19,20]. In general, bulge test experiments can be classified into three categories with respect to the shape of the tested membrane: circular, square, and rectangular. The plane-strain bulge test, which uses a rectangular membrane with aspect ratio larger than 4, results in a state that approximate plane strain is currently the most widely utilized bulge test for freestanding thin film testing, due to its unique advantages [12]: 1) its favorable application of micromachining routes, 2) a more accurate determination of elastic modulus when Poisson ratio is not exactly known and 3) an ideal technique for studying the plastic deformation behavior of thin films. But it should be noted that the plane-strain bulge analytical model was derived by assuming that a long rectangular

**Foundation item:** Project (11172258) supported by the National Natural Science Foundation of China
**Corresponding author:** Shi-guo LONG; Tel: +86-731-58228146; E-mail: longsg@xtu.edu.cn
DOI: 10.1016/S1003-6326(14)63466-X

membrane is in a perfectly plane-strain state. In fact, the resulting stress state in the film is quite complicated [21].

The accuracy and reliability of the bulge test have been analyzed by many researchers. PAN et al [22] verified analytical models of circular and square membrane by finite element analysis. By the same method, SMALL and NIX [23] systematically analyzed the influence of initial conditions such as residual stress and initial height of circular membrane, and then proposed a new approach to improve the accuracy of the spherical membrane bulge equations. VLASSAK [12] and HOHLFELDER [24] investigated the influence of bending stiffness on the deflection of a membrane and showed that bending moment can be ignored except for the edge of the membrane. However, these analyses focused on circular and square membrane bulge test rather than the plane-strain bulge test. Meanwhile, most current work emphasized the importance of controlling the experimental uncertainties instead of improving the accuracy of the method of data analysis in the plane-strain bulge test [17,18,25,26]. So, it is necessary to define a method which can analyze bulge test data to obtain more accurate results.

In this study, finite element method simulations are used to define virtual experiments, allowing a comparison between the actual mechanical properties of thin films and the mechanical properties calculated by the bulge analytical model (using the pressure−deflection data from the simulation). This provides a direct approach to verify the accuracy of the bulge equation in terms of our finite element results. The objectives of this research are to systematically investigate the effects of geometrics, initial condition (residual stress) and material properties on the existing bulge test model, and to define a method of data analysis to improve the accuracy of the plane-strain bulge model.

## 2 Bulge test principle

Approximated solutions using other different approaches for various membrane shapes have been derived by a number of researchers [18]. VLASSAK and NIX [3] derived an equation including the influence of residual stress to model the deformation of linear elastic rectangular membranes following an energy minimization approach originally developed by TIMOSHENKO [27]. In such a case, the deflection, $h$, at the center of a membrane of dimensions of $2a\times2b$ is a function of the applied pressure, $p$, the membrane geometry, and various material parameters:

$$h = f(p, E, v, \sigma_0, a, b, t) \quad (1)$$

where $E$ is the elastic modulus; $v$ is the Poisson ratio; $t$ is the thickness of the film; $\sigma_0$ is the residual stress of the film; $a$ and $b$ are the half width and the half length of the membrane, respectively. The dimensionless form of the above function is:

$$\frac{h}{a} = F\left(\frac{p}{E}, v, \frac{\sigma_0}{E}, \frac{b}{a}, \frac{t}{a}\right) \quad (2)$$

For a linear elastic membrane, this relationship can be approximated by the following expression:

$$p = C_1 \frac{\sigma_0}{a^2} h + C_2(\nu) \frac{Et}{(1-\nu\ )a^4} h^3 \quad (3)$$

where $C_1$ is a constant that depends on the membrane geometry; $C_2$ is a function of the membrane geometry and the Poisson ratio. Once the aspect ratio, $a/b$, of a rectangular membrane exceeds 4, the deflection at the center of the membrane is nearly independent of $a/b$ and can be approximated with the exact solution for an infinitely long rectangular membrane [3,4]. In other words, an infinitely long membrane is a good approximation for rectangular membranes with large $a/b$. As a result, the stress and strain are distributed uniformly across the width of the membrane, except for the outer edge, where the bending moment is significant. The exact solution for an infinitely long rectangular membrane can be obtained in this case, and Eq. (3) is then rewritten as follows:

$$p = 2\frac{\sigma_0}{a^2} h + \frac{4}{3}\frac{Mt}{a^4} h^3 \quad (4)$$

where $M=E/(1-v^2)$ is the plane-strain modulus. From the above equations, we can find that by plotting $p/h-h^2$ curve it will generate a straight line with a slope which relates to $M$ and the intercept that is proportional to the residual stress. Therefore, the thickness and width of the thin film are known, and both the plane-strain modulus and residual stress can be calculated readily by measuring the deflection versus pressure, and fitting the data to Eq. (4).

Nevertheless, it is important to point out that the real deformation of a rectangular membrane is commonly more complex than a hypothetical deformation, and the plane-strain state is only found near the center of the rectangular membrane. Consequently, the bulge equation is often applied to tests far less ideal [21,28]. In the case, the validity of the plane-strain bulge model will be analyzed by the finite element analysis in the rest of this paper.

## 3 Finite element model

In order to model the deformation behavior of thin films and verify the preceding analytical solutions of the plane-strain bulge test, finite element method simulations are carried out by using the commercial nonlinear finite element code ABAQUS [29]. Because of symmetry, only