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# Construction and solution of strain model along thickness of aluminum alloy plate under plastic deformation

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**Abstract:** A thickness strain model of aluminium alloy plate under plastic deformation, based on thin plate assumption was proposed. It is found that when ratio of stress fractions is constant during in-plane loading, ratios of strain components under various loading conditions are linearly related and these points of ratios form a $\eta-\eta$ line. Under these simple loadings, strains in thickness direction can be easily calculated by the $\eta-\eta$ line equation without integral and differential work. When the plate is under more complicated loading conditions, the thickness can be computed by the proposed optimization and piecewise calculation model. Validation computations indicate that the relative error of the results of the presented model is less than 0.75% compared with the proven theories and FE simulation. Therefore, the developed model can be applied to engineering calculation, e.g. pre-stretching analysis of aerospace aluminium thick plate, with acceptable accuracy.

**Key words:** isotropic linear hardening; thick plate; strain model; plastic deformation; aluminum alloy

## 1 Introduction

7000 series aluminum alloy thick plates are widely applied in aerospace for their combination of high strength, stress-corrosion-cracking resistance and toughness. Stretching is a typical process performed on solid solution treated aluminum thick plate to release quenching-caused residual stresses through leading uniform deformation to the plate in the rolling direction [1].

Since available and measurable variables in stretching process are strains of the plate, it is reasonable to establish the analytical model based on strain-space plasticity theory, which originated from soil and rock mechanics. DRUCKER [2] firstly considered the possibility of formulating the theory of elasto-plastic material in strain space. The studies performed by NAGHDI, TRAPP and CASEY [3,4] contributed to the establishment of the theory by using plastic strain as state variables with which most researchers are familiar. YODER and IWAN [5] employed the relaxation stress as state variables in formulations of strain space plasticity. HAN and CHEN [6] built strain-space plasticity formulation for hardening-softening materials with elasto-plastic coupling, while FARAHAT et al [7] modeled for concrete with compressive hardening-softening behavior in strain-space. LU and VAZIR [8] claimed that the stress- and strain-based plasticity theories are equivalent by offering the alternative conjugate expressions for the loading criteria, provided that the material laws used are identical in both approaches.

The tensile tests performed on AA7075-T6 and 2024-T3 by LEE and SHAUE [9] indicated that the materials are linear hardening. The compressive tests under quasi-static loading conducted by ABOTULA and CHALIVENDRA [10] and WU et al [11] showed the similar stress−strain relations. Although mechanical properties in the rolling and transverse directions of commercial wrought aluminum alloy are slightly different, the material can be considered to be isotropic for simplification of modeling.

The aluminum thick plate is in plane-stress state during stretching and in-plane stresses of it vary along

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the thickness-direction [1,12]. As is used in stress measurement methods, such as layer removal [13,14], incremental hole drilling [15], layer X-ray method [16] and crack compliance method [17], thick plate can be virtually divided into many thin sheets along its thickness-direction and stress evolution in the thick plate can be deduced from the stress status of the separated thin sheets by superposition which was described in Ref. [18]. Since the thickness of the virtually divided thin sheet in the thick plate cannot be measured directly, a model, which relates strain in the thickness-direction to strains in the rolling/transverse direction, is the basis for the research on residual stresses in the thick plate.

At present, the model for calculating thickness of metallic plate under in-plane deformation is still relatively lacking. Researches concerned with the evolution of plate's thickness are mostly focused on bending deformation. ZHU [19] studied large deformation pure bending of the wide plate made of the power-law-hardening material. The results showed that large curvature bending leads to a significant thickness reduction of the bent plate. COLLIE et al [20] used analytical models, numerical models and elastic–plastic FEA to predict the final deformed geometry of induction bends in thick-walled pipe. PENG et al [21] established a theoretical solution to thickness variation of bending metal sheet with perfect plasticity and linear hardening character. In general, researches concerning theory of plate/shell or non-linear plate theory [22] do not give solution of thickness-direction strain of thin plate under in-plane deformation and there are no models can be applied to calculating strain of thickness-direction $z$ for aluminum plate which is under in-plane deformation directly. With this model, the thickness of plate can be computed by knowing deformation in the rolling and the transverse directions.

## 2 Plate-layered hypotheses

Residual stresses in the aluminum thick plate is relieved by applying a uniform plastic strain on it [23] and a schematic diagram of stretching process for the plate is shown in Fig. 1. The plate is under uniaxial tension load which is uniformly distributed at each end of the plate along the rolling direction ($x$-direction) while quenching- caused residual stress $\sigma_r$, varies along the thickness- direction. Since the length and the width are much larger than the thickness, the thick plate is in-plane stress during the stretching process. In order to obtain stress state at a given depth, the thick plate is assumed to be made up of $N$ layers of thin plates and one of them is depicted in dash line in Fig. 1. The thin plate has a tiny thickness, e.g., less than 1/40 of thickness of the thick plate, so that the in-plane stress of it can be considered as constant throughout its thickness. By knowing the stress status of these thin plates, the stress evolution of the thick plate can be easily deduced.

The loading condition of the thin plate is also drawn in Fig. 1. The thin plate, with the same length and width as the thick plate, is under uniformly distributed loads, $F_x$ and $F_y$, in the rolling and the transverse directions respectively. $F_x$ and $F_y$ are caused by the material around the thin plate during stretching. With the increase of $F_x$ and $F_y$, stresses of the thin plate will grow from initial state to the stress of elastic limit $\sigma_x$. After that, the thin plate will deform plastically under further loading.

Figure 2 shows the evolution of the thin plate's strain state in the $x$ and $y$ directions during deformation. In the plot, the abscissa $\varepsilon_x$ is strain component in the rolling direction and the ordinate $\varepsilon_y$ is strain in the transverse direction. The straining path of the thin plate is nonlinear and drawn in solid line in Fig. 2. The

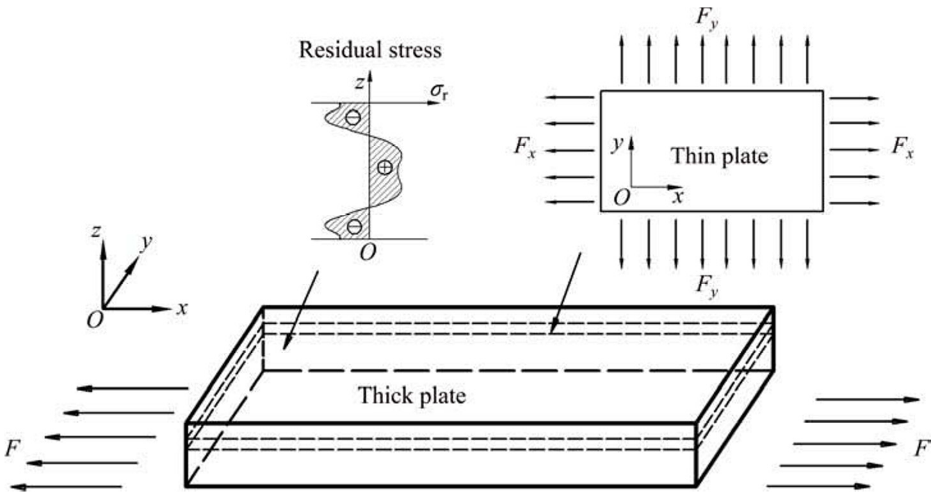


**Fig. 1** Schematic depiction of aluminium alloy plate to be stretched and thin plate under in-plane deformation