Materials Letters 73 (2012) 212–215

Contents lists available at SciVerse ScienceDirect

Materials Letters

journal homepage: www.elsevier.com/locate/matlet

Sesqui-power scaling of elasticity of closed-cell foams

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ARTICLE INFO

Article history:
Received 25 October 2011
Accepted 8 January 2012
Available online 13 January 2012

Keywords:
Elastic properties
Porous materials
Simulation and modeling
Closed-cell foams
Micromechanical modeling
Relative density

ABSTRACT

The scaling of Young's modulus of open-cell foam with the sesqui-power of the volume fraction of solid material phase (relative density), observed in experiments on expanded polystyrene foam, with volume fractions of the solid material phase ranging from 0.025 to 0.38, is rationed by employing the elastic solution for the compression of a thin spherical shell as a unit-cell think model for closed-cell foams. The resulting sesqui-power scaling is shown to adequately represent the behavior of other closed-cell foams (e.g. aluminum foams).

1. Introduction

Modeling of the elastic properties of closed-cell foams is an intensely discussed topic in the literature, see, e.g., [1–10]. Recently, classical schemes based on continuum micromechanics were used to model the elastic properties of highly-densified expanded polystyrene (HD-EPS) foam [11]. HD-EPS is a cellular material (see Fig. 1(a)) produced by densification of expanded polystyrene (EPS) through autoclavic curing. The so-obtained porous material is characterized by densities greater than the density of the source material EPS ($\rho_{\rm eff}$~25 kg/m^3). Fig. 1(b) shows Young's modulus obtained from compression tests on the source material and on HD-EPS with $\rho_{\rm eff}$~100 and ~400 kg/m^3, respectively. When underlying a density of $\rho_{\rm m}=1050$ kg/m^3 of the solid material phase in the composite (polystyrene, according to DIN 7741), the volume fraction of the solid material phase, $f_{\rm m}=\rho_{\rm eff}/\rho_{\rm m}$, is given as ~0.025 for the source material, and ~0.095 and ~0.38, respectively, for HD-EPS.

For modeling based on the differential scheme, which emerged as the most suitable homogenization method for HD-EPS in the framework of continuum micromechanics [11][1] (see Fig. 1(b)), the elastic properties of the solid material phase (polystyrene) were set to: Young's modulus $E_{\rm m}=3.35$ GPa, Poisson's ratio $\nu_{\rm m}=1/3$ [according to DIN 7741]. The prediction of the differential scheme for Young's modulus almost coincides with the scaling relation given in [1,12] for bending-dominated behavior of (open-cell) foams, $E_{\rm eff}/E_{\rm m}\propto(\rho_{\rm eff}/\rho_{\rm m})^2=f_{\rm m}^2$, what leads – with a proportionality factor of one [1,12] – to $E_{\rm eff}=E_{\rm m}f_{\rm m}^2$ (see Fig. 1(b)).

Upon a close inspection, the representation of the experimental data, with a slope of 2 according to the differential scheme *and* the scaling relation given in [1,12] in a double-logarithmic diagram (see Fig. 1(c)), is less accurate for small values of the solid-material fraction $f_{\rm m}$. The experimental data rather follows a scaling in the form $E_{\rm eff}\propto E_{\rm m}f_{\rm m}^{3/2}$, i.e., sesqui-power scaling. Roughly, the same sesqui-power scaling applies to the proportionality limit stress $\bar{\sigma}_{\rm eff}$ in the stress–strain relation of HD-EPS (see Fig. 1(c)). In this paper, the experimentally-observed scaling is rationed by employing the elastic solution for the compression of a thin spherical shell as a unit-cell think model for closed-cell foams.

2. Modeling

In [13,14], the solution for the contact problem depicted in Fig. 2(a) is derived: compressive loading of a thin spherical shell (radius R and thickness h) characterized by Young's modulus $E_{\rm m}$ in frictionless contact with a rigid plane. Depending on the value of the applied load F, two axisymmetric deformation states (with δ denoting the apex deformation) of the shell were obtained in [13,14]:

- Configuration I: for low load levels, the shell flattens against the horizontal plane with a formation of a circular fold (see Fig. 3(a));
- Configuration II: for higher load levels, the flattened region buckles inward (towards the center of the shell) with an inversion of the curvature (with respect to the undeformed configuration) and an increase of the circular fold (see Fig. 3(a)).

The elastic energies U associated with these two deformation states were obtained as [13,14]:

$$U_{\rm I}=\frac{C_0}{4}\frac{E_{\rm m}h^{5/2}}{R}\delta^{3/2}+C_1\frac{E_{\rm m}h}{R}\delta^3 \tag{1}$$

and

$$U_{\rm II}=C_0\frac{E_{\rm m}h^{5/2}}{R}\delta^{3/2}+C_2\frac{E_{\rm m}h^3}{R}\delta, \tag{2}$$

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[1] See [5] for a similar study dealing with modeling of aluminum foam properties based on homogenization schemes from continuum micromechanics.

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doi:10.1016/j.matlet.2012.01.038

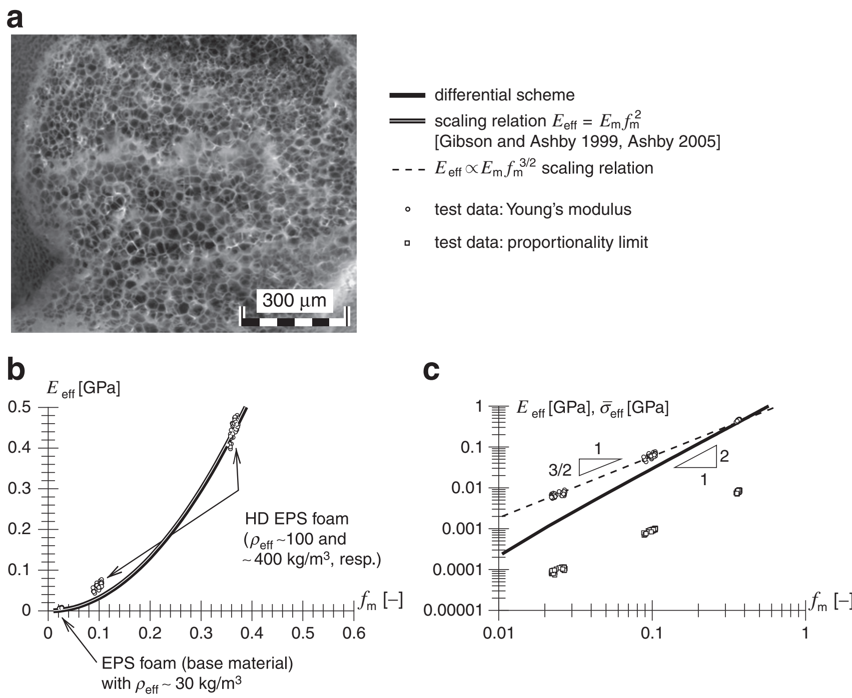

Fig. 1. (a) ESEM micrograph of HD-EPS with $\rho_{eff} = 112$ kg/m³ [12], (b) modeling of effective Young's modulus E_{eff} of HD-EPS foam by the differential scheme (homogenization scheme based on continuum micromechanics) as a function of the solid-material fraction f_m [11], and (c) sequi-power scaling of E_{eff} and proportionality limit $\bar{\sigma}_{eff}$ of HD-EPS foam.

with constants C_0, C_1, and C_2. In Eq. (1), the first term stems from the existence of an axisymmetric, circular fold, while the second term is associated with compressive loading of the flattened-out portion of the spherical shell. Similarly, the axisymmetric fold in Configuration II entails the first term of U_{II}, while the second term represents the deformation state in the buckled region of the spherical shell [13,14]. The forces associated to the energy terms U_I and U_{II}, $F = \partial U/\partial\delta$, follow as [13,14]

$$F_I = \frac{\partial U_I}{\partial \delta} = \frac{3C_0}{8}\frac{E_m h^{5/2}}{R}\delta^{1/2} + 3C_1\frac{E_m h}{R}\delta^2, \tag{3}$$

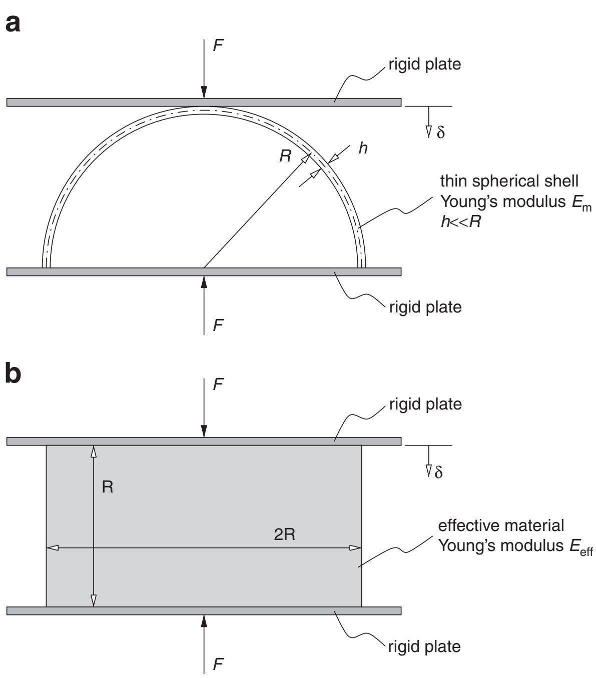

Fig. 2. (a) Contact problem of elastic shperical shell [14,15]; (b) unit-cell think model for closed-cell foams with effective material properties.

$$F_{II} = \frac{\partial U_{II}}{\partial \delta} = \frac{3C_0}{2}\frac{E_m h^{5/2}}{R}\delta^{1/2} + C_2\frac{E_m h^3}{R}. \tag{4}$$

Taking the considered spherical half-shell as unit-cell think model for closed cell foams under uniaxial loading (see Fig. 2), with a volume of the unit cell of $V = \text{area} \times \text{height} = (2R)^2 \times R = 4R^3$, and underlying that the volume fraction of solid phase in the foam material f_m is controlled by the sphere radius R, i.e., h remains constant, the volume fraction of the solid material is given as

$$f_m = \frac{V_m}{V} = \frac{2\pi R^2 h}{4R^3} = \frac{\pi}{2}\frac{h}{R}, \tag{5}$$

where the volume V_m of the thin half-shell ($h \ll R$) is determined from its surface $2\pi R^2$.

The deformation δ of the sought-for effective material under the same axial compressive loading conditions (see Fig. 2(b)) is related to the effective Young's modulus E_{eff} as

$$\delta = \varepsilon R = \frac{\sigma}{E_{eff}}R = \frac{F}{(2R)^2 E_{eff}}R = \frac{F}{4RE_{eff}}, \tag{6}$$

where the uniaxial stress and strain in the direction of loading, $\sigma = F/(2R)^2$ and $\varepsilon = \delta/R$, respectively, were used.

Restricting further considerations to low load levels (i.e., using Eq. (3)) and

- assuming that the first term on the r.h.s. of Eq. (3) is controlling the material behavior (for small deformations: $C_0/8(\delta/h)^{1/2} \gg C_1(\delta/h)^2$, see Appendix **A**), inserting Eq. (6) into Eq. (3) gives

$$F_I = \frac{3C_0}{8}\frac{E_m h^{5/2}}{R}\frac{F_I^{1/2}}{2R^{1/2}E_{eff}^{1/2}}. \tag{7}$$

Replacing R by $\pi h/(2f_m)$ (see Eq. (5)) and arranging constant parameter at the l.h.s. give access to dimensionless quantities on both sides:

$$\frac{16}{3C_0}\left(\frac{\pi}{2}\right)^{3/2}\left(\frac{F_I}{E_m h^2}\right)^{1/2} = \left(\frac{E_m}{E_{eff}}\right)^{1/2} f_m^{3/2} = \text{constant}, \tag{8}$$