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On the transverse cracking and stiffness degradation of aged angle-ply laminates

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Abstract

A modified shear lag analysis, taking into account the concept of stress perturbation function, is employed to evaluate the effect of transverse cracks on the stiffness reduction in aged angle-ply laminated composites. The results of this paper represent well the dependence of the degradation of elastic properties on the cracks density, hygrothermal conditions and the fibre orientation of the outer layers.

Keywords: Hygrothermal effect; Transverse cracking; Angle-ply laminates; Stress perturbation function; Stiffness reduction

1. Introduction

Laminated composite materials are most suitable for structural applications where high strength-to-weight and stiffness-to-weight ratios are required. However, insufficient knowledge of both static strength and life expectancy still limits the number of working parts where such materials can be safely used. In most practical situations, the first observed damage mechanism is matrix cracking. A careful investigation of this phenomenon is necessary, as it is usually followed by other more harmful damage modes, which can entail the failure of the whole structure. Numerous approaches and analytical models have been developed for predicting the laminate degradation. The shear lag analysis was used to predict the stiffness reduction according to the transverse crack density [1–7]. A variational approach was used by Hashin [8] to study the elastic property degradation and the stress distribution in a cracked cross-ply laminate.

On the other hand, in polymer composite, the matrix is very sensitive to the variation of temperature and moisture ratio. Indeed, at high temperature and moisture concentration, we have a considerable degradation of the matrix that reduces the mechanical characteristics of the laminate and the strength failure of material [9–13].

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In this paper, a progressive shear–lag model [3] is used with some modifications to predict the effect of transverse cracks on the stiffness degradation of hygrothermal aged angle-ply composite laminates. General expression for longitudinal modulus reduction versus transverse crack density is obtained by introducing the stress perturbation function [4,5,7]. Good agreement is obtained comparing prediction with experimental results. Then, the hygrothermal effect on the material properties of the laminate is taken into account to evaluate the stiffness loss in angle-ply laminates containing transverse cracks. The obtained results illustrate well the dependence of the degradation of elastic properties on the cracks density, hygrothermal conditions and the fibre orientation of the outer layers.

2. Theoretical modeling

It is well known in many studies [5–7,9–13] that the material properties are function of temperature and moisture. In terms of a micro-mechanical model of laminate, the material properties may be written as [14]

$$E_{\mathrm{L}} = V_{\mathrm{f}} E_{\mathrm{f}} + V_{\mathrm{m}} E_{\mathrm{m}} \tag{1}$$

$$\frac{1}{E_{\mathrm{T}}} = \frac{V_{\mathrm{f}}}{E_{\mathrm{f}}} + \frac{V_{\mathrm{m}}}{E_{\mathrm{m}}} - V_{\mathrm{f}} V_{\mathrm{m}} \frac{v_{\mathrm{f}}^2 \left(\frac{E_{\mathrm{m}}}{E_{\mathrm{f}}}\right) + v_{\mathrm{m}}^2 \left(\frac{E_{\mathrm{f}}}{E_{\mathrm{m}}}\right) - 2 v_{\mathrm{f}} v_{\mathrm{m}}}{V_{\mathrm{f}} E_{\mathrm{f}} + V_{\mathrm{m}} E_{\mathrm{m}}} \tag{2}$$

$$\frac{1}{G_{\mathrm{LT}}}=\frac{V_{\mathrm{f}}}{G_{\mathrm{f}}}+\frac{V_{\mathrm{m}}}{G_{\mathrm{m}}} \tag{3}$$

$$V_{\mathrm{LT}}=V_{\mathrm{f}}v_{\mathrm{f}}+V_{\mathrm{m}}v_{\mathrm{m}} \tag{4}$$

In the above equations, V_{f} and V_{m} are the fibre and matrix volume fractions and are related by

$$V_{\mathrm{f}}+V_{\mathrm{m}}=1 \tag{5}$$

E_{f}, G_{f} and v_{f} are the Young's modulus, shear modulus and Poisson's ratio, respectively, on the fibre, and E_{m}, G_{m} and v_{m} are corresponding properties for the matrix.

It is assumed that E_{m} is a function of temperature and moisture, as is shown in Section (3.2), then E_{L}, E_{T} and G_{LT} are also functions of temperature and moisture.

2.1. Stiffness reduction model in the angle-ply laminates

Transverse matrix cracking is a common damage mode in angle-ply laminates under uniaxial tension. The matrix cracks develop in the fibre direction and extend across the 90°-ply width from the free edges.

For ideally equidistanced crack spacing in 90° layers for symmetric and balanced laminates, Tounsi and co-workers [5,7] and Joffe and Varna [4] showed using the stiffness reduction model that the crack spacing $2l_0$ (Fig. 1) reduces the extensional stiffness of the specific composite laminates according to

$$\frac{E_x}{E_{x0}}=\frac{1}{1+a\bar{\rho}R(\bar{l}_0)} \tag{6}$$

Where $\bar{\rho}=\frac{1}{2\bar{l}_0}\left(\bar{l}_0=\frac{l_0}{t_{90}}\right)$ is normalised crack density and a is a known function, dependent on elastic properties and geometry of $\theta°$ and 90° layer:

$$a=\frac{E_{90}t_{90}}{E_x^{\theta}t_{\theta}}\left(\frac{1-v_{12}v_{xy}^{0}}{1-v_{12}v_{21}}\right)\left(1+\upsilon_{xy}^{\theta}\frac{S_{xy}^{\theta}t_{90}+S_{12}t_{\theta}}{S_{yy}^{\theta}t_{90}+S_{11}t_{\theta}}\right) \tag{7}$$

E_x^{θ} and E_{90} are the Young's moduli of $\theta°$ and 90° layers, respectively. v_{xy}^{0} is the Poisson's ratio of the undamaged laminate.

The unknown function $R(\bar{l}_0)$ is a function of the ideal crack spacing, which influences the elastic constant reduction rate, and has different forms according to the stiffness analysis model adopted such as shear–lag models or variational models.

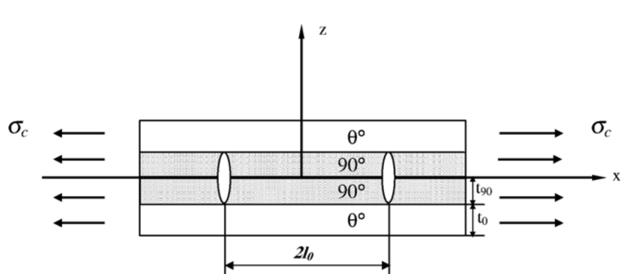

Fig. 1. Transverse cracked angle-ply laminate and geometric model.

2.2. Computation of the stress perturbation function using shear lag model

We have used in this study the model developed by Berthelot [3]. This latter is modified by introducing the stress perturbation function. It can be shown that the stress perturbation function $R(\bar{l}_0)$ is found in a following form

$$R(\bar{l}_0)=\int_{-\bar{l}_0}^{+\bar{l}_0}\frac{\cosh(\xi\,\bar{x})}{\cosh(\xi\bar{l}_0)}\mathrm{d}\,\bar{x}=\frac{2}{\xi}\tanh(\xi\bar{l}_0), \tag{8}$$

Where ξ is the shear lag parameter

$$\xi^2=\bar{G}\,\frac{t_{90}(t_{90}E_{90}+t_{\theta}E_x^{\theta})}{t_{\theta}E_x^{\theta}E_{90}} \tag{9}$$

The coefficient $\bar{G}$ depends on used assumptions about the longitudinal displacements and shear stress distribution:

- Firstly, assumptions on the longitudinal displacements: the variation of the longitudinal displacement is supposed to be parabolic in thickness of 90° layer:

$$u_{90}(x,z)=\bar{u}_{90}(x)+\left(z^2-\frac{t_{90}^2}{3}\right)A_{90}(x); \tag{10}$$

the variation of the longitudinal displacement is to be determined in thickness of $\theta°$ layers:

$$u_{\theta}(x,z)=\bar{u}_{\theta}(x)+f(z)A_{\theta}(x). \tag{11}$$

- Secondly, assumptions on the shear stresses, similar in $\theta°$ and 90° layers, which can be obtained by assuming that the transverse displacement is independent of the longitudinal coordinate:

$$\sigma_{xz}^{i}=G_{xz}^{i}\gamma_{xz}^{i}$$

$$\gamma_{xz}^{i}=\frac{\partial u_i}{\partial z}+\frac{\partial w_i}{\partial x}\approx\frac{\partial u_i}{\partial z} \tag{12}$$

$i=\theta°, 90°$

The coefficient $\bar{G}$ is done by:

$$\bar{G}=\frac{3G}{t_{90}} \tag{13}$$

The generalized shear modulus of the elementary cell:

$$G=\frac{G_{xz}^{90}}{1-3\dfrac{G_{xz}^{90}}{G_{xz}^{\theta}}\dfrac{f(t_{90})}{t_{90}f'(t_{90})}} \tag{14}$$

An analytical function of the variation function was considered [3]:

$$f(z)=\frac{\sinh\frac{t_{\theta}}{t_{90}}\eta_t}{\frac{t_{\theta}}{t_{90}}\eta_t}-\cosh\eta_t\left(1+\frac{t_{\theta}}{t_{90}}-\frac{z}{t_{90}}\right); \tag{15}$$