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# A compensation of Young's modulus in polysilicon structure with discontinuous material distribution

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**Abstract**

This paper is concerned with the effect of material discontinuities, such as porosity, on the modulus of elasticity and attempts to simulate the effects of porosity in polysilicon using a finite element model. The proposed Young's modulus model is applied to the simulation of a polysilicon MEMS gyro. When the compensated Young's modulus is used in this simulation, we can reduce the difference between the simulation and the actual experimental result.


*Keywords:* Microstructure; Material discontinuity; Porosity; Porous elasticity modulus; Finite element model

Material discontinuities such as porosities and unevenness exist in almost all materials to some extent. Therefore, the effects of porosities should be concerned to any design in the microstructure. Since these material discontinuities are randomly and unevenly distributed, it is very difficult to predict the material properties and behaviors, uniformly.

This paper is concerned primarily with the effects of material discontinuities such as porosities on the modulus of elasticity and attempts to simulate the finite element model of microstructure as the effects of porosities in the material. Instead of the random and uneven distributions of porosities, the approximate pore size and distribution are used for the approximate modulus of elasticity. This model uses linear elastic deformations but can be extended to the nonlinear and plastic deformations. Compensated Young's modulus with proposed method is verified with simulation and experiment of measuring the natural frequency of MEMS gyro sensor.

The material properties with porosity in the mechanical structure can be given by,

$$E = x^n E_0 \quad (1)$$

where $E$ is the elastic modulus with porosity, $E_0$ is the elastic modulus without porosity, $x$ is the volume density and $n$ is the density index [1,2].

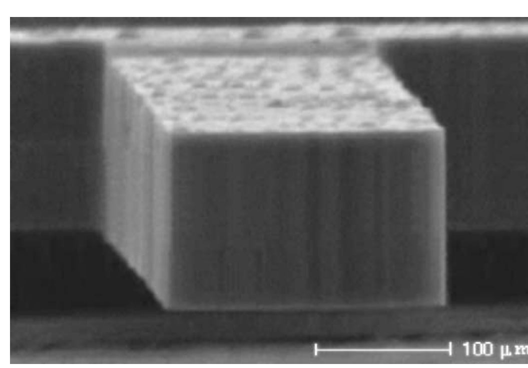


Fig. 1. The SEM picture of polysilicon-fabricated cantilever microstructure with material discontinuities (pore). Young's modulus compensation parameter is measured with this cantilever microstructure.

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Table 1
Elastic modulus of polysilicon microstructure

| Volume porosity ($\bar{V}$, %) | Pore average size (diameter, μm) | $E_0$ (GPa) | $E$ (GPa) |
|---|---|---|---|
| 1.7 | 2.0 | 179 | 173 |
| 5.4 | 5.0 | | 157 |
| 6.0 | 8.0 | | 155 |
| 10.5 | 10.0 | | 141 |

Table 2
Slope for pore sizes

| Pore size (μm) | Slope, α |
|---|---|
| 2.0 | 2.103 |
| 5.0 | 2.247 |
| 8.0 | 2.201 |
| 10.0 | 2.048 |

The structural rigidity mechanically depends on the relationship of stress-strain ($\{\sigma\}=[D]\{\varepsilon\}$) and the elastic constants $[D]$ of 2-D and 3-D element for an isotropic material are given by [3].

For the plane stress,

$$D(x) = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & (1-v)/2 \end{bmatrix} \tag{2}$$

For the plane strain,

$$D(x) = \frac{E}{(1+v)(1-2v)}\begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & (1-2v)/2 \end{bmatrix} \tag{3}$$

For the solid element,

$$D(x) = \frac{E(1-v)}{(1+v)(1-2v)} \times \begin{bmatrix} 1-v & \frac{v}{1-v} & \frac{v}{1-v} & 0 & 0 & 0 \\ \frac{v}{1-v} & 1 & \frac{v}{1-v} & 0 & 0 & 0 \\ \frac{v}{1-v} & \frac{v}{1-v} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2v}{2(1-v)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2v}{2(1-v)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2v}{2(1-v)} \end{bmatrix}. \tag{4}$$

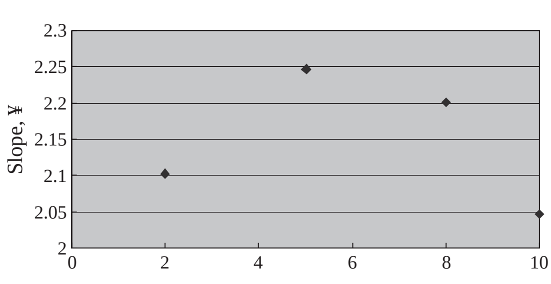


Fig. 2. Relationship between slope in Eq. (8) and pore size.

Table 3
Experiment constants of compensated elastic modulus equation

| Constant | $p_1$ | $p_2$ | $p_3$ |
|---|---|---|---|
| Value | 0.00386 | −5.988 | 0.4275 |

The variation of structural rigidity with respect to material porosity can be calculated using the following relationships.

For the static problem,

$$[K]\{u\} = \{F\}$$

$$[K]\frac{\partial\{u\}}{\partial x} = -\frac{\partial[K]}{\partial x}\{u\} = \{\tilde{f}\} \tag{5}$$

where $\{\tilde{f}\}$ is the pseudo-load.

For the eigenvalue problem,

$$[K]\{\phi\} - \lambda[M]\{\phi\} = 0$$

$$\frac{\partial\lambda_i}{\partial x} = \frac{\{\phi_i\}^T\left(\frac{\partial[K]}{\partial x} - \lambda_i\frac{\partial[M]}{\partial x}\right)\{\phi_i\}}{\{\phi_i\}^T[M]\{\phi_i\}}$$

$$= \{\phi_i\}^T\frac{\partial[K]}{\partial x}\{\phi_i\} - \lambda_i\{\phi_i\}^T\frac{\partial[M]}{\partial x}\{\phi_i\}$$

$$= \frac{E'(x)}{E(x)}\{\phi_i\}^T[K]\{\phi_i\} - \frac{1}{x}\lambda_i\{\phi_i\}^T[M]\{\phi_i\} \tag{6}$$

where $\{\phi_i\}^T[M]\{\phi_i\}=1$.

The entries of stiffness matrix $[K]$ can be written by Eq. (7).

$$[K] = \int_V [B]^T[D(V)][B]\mathrm{d}V \tag{7}$$

where $[B]$ spatial derivative matrix of displacement variables.

Porosity can be defined as the fraction of void volume in the material [4–6]. The effects of porosity on Young's modulus have dealt with fitting an empirical curve to actual

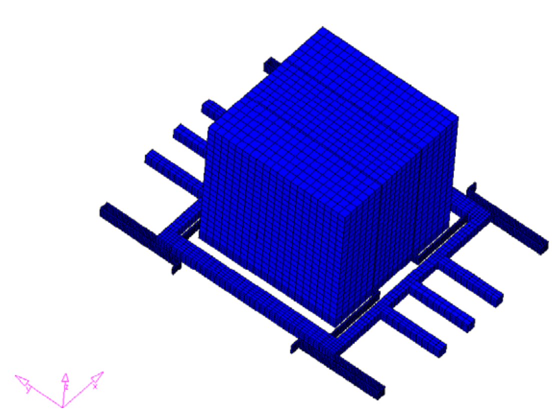

Fig. 3. MEMS gyro FEM model.