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Process Control

Consistency and Asymptotic Property of a Weighted Least Squares Method for Networked Control Systems ☆

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ABSTRACT

In this paper, we study the problems related to parameter estimation of a single-input and single-output networked control system, which contains possible network-induced delays and packet dropout in both of sensor-to-controller path and controller-to-actuator path. A weighted least squares (WLS) method is designed to estimate the parameters of plant, which could overcome the data uncertainty problem caused by delays and dropout. This WLS method is proved to be consistent and has a good asymptotic property. Simulation examples are given to validate the results.

1. Introduction

In a networked control system (NCS), there may exist random network-induced delays and packet dropout in the sensor-to-controller (S-C) path and the controller-to-actuator (C-A) path due to the communication access constraints [1,2]. Network-induced delays and packet dropout could cause data uncertainty problems and bring difficulties for system identification, control, or fault detection.

Problems related to NCS identification have drawn more and more attentions in recent years. Particularly, Fei *et al.* [3] utilized a discard-packet strategy, a cubic spline interpolation, and buffers to the actuator to overcome the data uncertainty problem in an NCS and proposed a recursive estimation method. Wang *et al.* [4] formulated an NCS in a continuous-time system with non-uniformly non-synchronized sampled data, and proposed a modified version of the simplified refined instrumental variable method to identify the parameter offline. Liu and Wang [5] extended the results [4] to the case with colored noise. Shi *et al.* [6] gave a recursive parameter estimator for closed-loop system with randomly missing output data. Shi and Fang [7] proposed a recursive method for open-loop system with randomly missing measurements of plant's input and output.

In our previous work [8], we have proved that the data set of a single-input and single-output (SISO) NCS with delays and dropout in both of the S-C and C-A paths is *informative* under very weak conditions, but we did not design an identification method. The result [8] is useful to design a consistent parameter estimator for the NCS in this paper, since the setup of NCSs is the same and the *informative* data set is a necessary condition for consistent parameter estimation.

The results in Refs. [3–7] cannot be applied to the NCS in this paper due to the following four reasons: (1) the M sequence used as the control signal [3] cannot be generated by feedback controllers; (2) with the remote computers used in closed loop to generate control signals [4,5], and those computers' outputs are required to be independent of their inputs, which could not be achieved by feedback controllers; (3) an adaptive controller instead of a linear time-invariant (LTI) one was required [6]; and (4) the problem in Ref. [7] was for open-loop setup.

In this paper, we consider system identification of a SISO NCS with a common setup, which includes an LTI plant, an LTI controller, and network transmissions in both the S-C and C-A paths that contain random delays and dropout. Motivated by Isaksson's work [11], in which a modification idea for the standard least squares (LS) method was proposed to estimate the parameter of an open-loop system with randomly missing output data, we design a weighted least squares (WLS) method. This WLS method could overcome the data uncertainty problems caused by random delays and dropout. Based on our previous result [8], we prove that this WLS method is consistent, *i.e.* its parameter estimate converges to the "true" value [9], and has a good asymptotic property in the sense that the product of estimation error and square root of data length converges to a Gaussian distribution, which means that the estimation error decays with the reciprocal of the square root of data length [10].

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2. NCS Setup and Problem Statement

Notation 1. Throughout the paper, "q", "$\bar{\mathrm{E}}$", and "Pr" represent "forward shift operator, *i.e.* $qx_k = x_{k+1}$", "a symbol introduced by $\bar{\mathrm{E}}x_k = \lim_{L\to\infty}\frac{1}{L}\sum_{k=1}^{L}\mathrm{E}x_k$ in Ref. [10], where $\{x_k\}$ is a sequence of quasi-stationary signal", and "probability", respectively.

The SISO NCS considered is shown in Fig. 1, where the reference input is zero, and the network-induced delays and packet dropout may occur randomly in both of the S-C and C-A paths. We assume that the actuator and the sensor are clock driven with a fixed sampling interval.

2.1. Closed-loop model

The LTI plant and the LTI controller are described by

$$\begin{aligned} y_k &= G\left(q^{-1}\right)u_k + H\left(q^{-1}\right)e_k \\ u_k^{\mathrm{c}} &= F\left(q^{-1}\right)y_k^{\mathrm{c}}. \end{aligned} \tag{1}$$

where $y_k \in \boldsymbol{R}$ and $u_k \in \boldsymbol{R}$ are the plant's output and input at time instant k, $y_k^{\mathrm{c}} \in \boldsymbol{R}$ and $u_k^{\mathrm{c}} \in \boldsymbol{R}$ are the controller's input and output at k, and $\{e_k \in \boldsymbol{R}\}$ is a sequence of independent and identically distributed (i.i.d.) random signals with zero mean values, variances λ^2, and bounded moments of order $4+\delta$ with some $\delta > 0$ [10], respectively.

The structure of the plant in Eq. (1) is assumed to be autoregressive exogenous (ARX), *i.e.*, $G(q^{-1}) \triangleq B_0(q^{-1})/A_0(q^{-1})$ and $H(q^{-1}) \triangleq 1/A_0(q^{-1})$ with $B_0(q^{-1}) \triangleq b_1^0 \cdot q^{-1} + \cdots + b_{n_b}^0 \cdot q^{-n_b}$ and $A_0(q^{-1}) \triangleq 1 + a_1^0 \cdot q^{-1} + \cdots + a_{n_a}^0 \cdot q^{-n_a}$. For the polynomial orders of $A_0(q^{-1})$ and $B_0(q^{-1})$, n_a and n_b, we assume that $n_a \ge n_b$, without loss of generality.

The parameter to be estimated is

$$\boldsymbol{\theta}_0 \triangleq \left[a_1^0, \ \cdots, \ a_{n_a}^0, \ b_1^0, \ \cdots, \ b_{n_b}^0\right]^{\mathrm{T}}.$$

The candidate parameter space used for estimation is denoted as $D_{\mathrm{M}} \triangleq \{\boldsymbol{\theta}_i\} \subset \boldsymbol{R}^{n_a+n_b}$ [10], where

$$\boldsymbol{\theta}_i \triangleq \left[a_1^i, \ \cdots, \ a_{n_a}^i, \ b_1^i, \ \cdots, \ b_{n_b}^i\right]^{\mathrm{T}}. \tag{2}$$

It is assumed that $\forall \boldsymbol{\theta}_i \in D_{\mathrm{M}}$, $\|\boldsymbol{\theta}_i\|_2$ is bounded.

Remark 1. According to Refs. [6] and [7], we assume that the polynomial orders of the ARX plant, n_a and n_b, are known, since they can be determined by using the statistical F-test [12–14] or the Akaike information criterion [15].

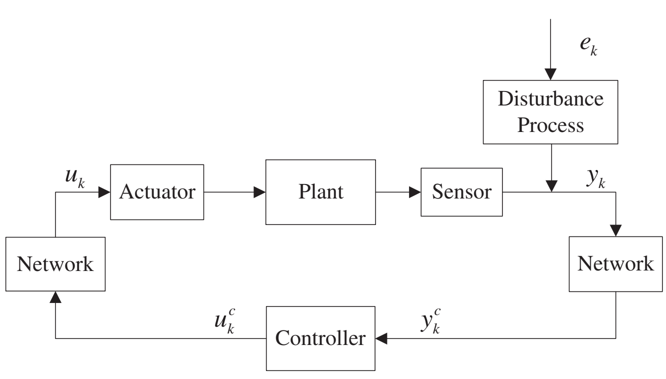

Fig. 1. SISO NCS.

2.2. Network transmission

The maximal steps of possible delays in the S-C and C-A paths are assumed to be $\tau_{\max}^{\mathrm{sc}}$ and $\tau_{\max}^{\mathrm{ca}}$ steps, respectively. Then y_k and u_k^{c} with delays longer than $\tau_{\max}^{\mathrm{sc}}$ and $\tau_{\max}^{\mathrm{ca}}$ steps will be discarded when they finally arrives.

Notation 2. σ_k^y and $\sigma_k^{u^c}$ denote the transmission states of y_k and u_k^{c} at time instant k, respectively,

$$\sigma_k^y\left(\text{or } \sigma_k^{u^c}\right) = \begin{cases} -1, & \text{if } y_k\left(\text{or } u_k^{\mathrm{c}}\right) \text{is dropped;} \\ 0, & \text{if } y_k\left(\text{or } u_k^{\mathrm{c}}\right) \text{is delivered successfully;} \\ i, & \text{if } y_k\left(\text{or } u_k^{\mathrm{c}}\right) \text{suffers } i\text{-step delays,} \\ & 1 \le i \le \tau_{\max}^{\mathrm{sc}}\left(\text{or } 1 \le i \le \tau_{\max}^{\mathrm{ca}}\right). \end{cases}$$

Remark 2. According to Refs. [1] and [2], it is common to assume that the delays and dropout in network transmission satisfy Bernoulli distribution. Therefore, $\{\sigma_k^y\}$ and $\{\sigma_k^{u^c}\}$ are two Bernoulli processes.

If $\sigma_k^y \ne 0$ (or $\sigma_k^{u^c} \ne 0$), *i.e.* y_k (or u_k^{c}) is not available to the controller (or the actuator) at time instant k, multiple update mechanisms can be used by the controller (or the actuator) to update y_k^{c} (or u_k), such as "0", "latest packet in the buffer", or "previous step value" update mechanisms [1,2]. In this paper, for the sake of generalization, we do not make any assumption on the update mechanisms adopted by the controller and the actuator.

2.3. Recovery of data set

Due to the influences of delays and dropout in the S-C and C-A paths, the plant's output may be received disorderly or lost on the controller side, and it is uncertain which packet sent from the controller is used by the plant. These data uncertainty problems bring difficulties for parameter estimation.

Fortunately, some techniques about NCS have been provided for data recovery on the controller side, such as the sequence numbering technique [4] and the smart sensor technology [6,16–19]. By using the sequence numbers of the packets received by the controller, the disorder problem caused by delays in the S-C path can be solved after $\tau_{\max}^{\mathrm{sc}}$ time instants at most; using the smart sensor technology (*i.e.* the actuator feedbacks the sequence number of plant's input to controller by sending it to the sensor and further adding it to the packet transmitted by the sensor), we could verify the packet used by the plant.

We make following assumption to recover the plant's input and output data on the controller side.

Assumption 1. The sequence numbering technique [4] and the smart sensor technology [6,16–19] are used for data recovery on the controller side. Except for the influence of dropout on the S-C path, all the other data uncertainties caused by unreliable transmission can be recovered. Then the data set $Z_{\mathrm{id}}^L \triangleq \{\bar{y}_1, \ u_1, \ \cdots, \ \bar{y}_L, \ u_L\}$ is available on the controller side for parameter estimation at time instant $L + \max\{\tau_{\max}^{\mathrm{sc}}, \tau_{\max}^{\mathrm{ca}}\}$, where

$$\bar{y}_k = \begin{cases} y_k, & \text{if } \sigma_k^y \ne -1 \\ 0, & \text{if } \sigma_k^y = -1. \end{cases} \tag{3}$$

Compared with the plant's input and output data set, $Z_{\mathrm{p}}^L \triangleq \{y_1, u_1, \cdots, y_L, u_L\}$, Z_{id}^L is obviously incomplete lacking of dropped plant's output data.

2.4. Formulation

We also have following assumption on the NCS.

Assumption 2.

(1) Delays and dropout occur independently of $\{e_k\}$;