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Process Model

Identification of LPV Models with Non-uniformly Spaced Operating Points by Using Asymmetric Gaussian Weights☆

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ABSTRACT

In this paper, asymmetric Gaussian weighting functions are introduced for the identification of linear parameter varying systems by utilizing an input–output multi-model structure. It is not required to select operating points with uniform spacing and more flexibility is achieved. To verify the effectiveness of the proposed approach, several weighting functions, including linear, Gaussian and asymmetric Gaussian weighting functions, are evaluated and compared. It is demonstrated through simulations with a continuous stirred tank reactor model that the proposed approach provides more satisfactory approximation.

1. Introduction

Modern industrial processes are operated over a wide operating range and often display strong static and dynamic nonlinearities. The traditional linear models can no longer meet the requirement for model-based control. Accordingly, finding a sound and low cost nonlinear identification approach to approximate nonlinear processes over a broad operating regime is crucial and indispensable [1].

In the identification of nonlinear systems, several black box modeling approaches characterized by the usage of theoretically sound nonlinear functions such as nonlinear AR(MA)X [2], artificial neural network models [3], and blocked-oriented models such as Hammerstein and Wiener models [4] have been studied. Since these models have complex structure and need difficult computation, their applications to industrial processes are limited.

Recently, linear parameter varying (LPV) model identification has attracted great attention from academia and industry [5]. The terminology of LPV was first introduced by Shamma and Athans [6] in the study of gain scheduling control. The study on LPV systems has been extended to the theory of linear systems [1]. Much work has been on the identification of LPV systems [7,8]. LPV approaches are also applied to aerospace systems including high performance aircraft, missiles and turbofan engines [9].

Most available references on input–output LPV are based on parameter interpolation, assuming that the scheduling parameter varies continuously [10]. However, nonlinear functions are complex in the denominator of transfer function, which may cause numerical problems during model identification [10]. Besides, the input excitation signal for this representation causes too much upset, which may be costly or even unrealistic in practice [11]. To circumvent these difficulties, a multi-model LPV model is proposed by interpolating local linear models [10]. With local linear models and model interpolation philosophy, the identification method is relatively simple and the stability of this LPV models is guaranteed [12].

Essentially, proper weighting functions are required to combine local linear models into a global LPV model in the multi-model LPV structure. The available options are linear weight function [10] and Gaussian weight function [12]. The linear weight functions can be used conveniently, but it is not sufficient to capture the full dynamic behavior of a nonlinear process. Owing to relative small number of parameters and superior performance, Gaussian weighting functions have been widely adopted in the multi-model structures [13] and fuzzy sets [14]. A drawback of Gaussian weighting functions is that the operating points for local linear models should have an equal distance, which limits their feasibility and causes large inconvenience in applications.

To overcome the disadvantage of Gaussian weights, asymmetric Gaussian weighting function is introduced for the identification of multi-model LPV structure in this paper. The locations of operating points can be selected freely. Linear, Gaussian and asymmetric Gaussian functions are used and compared in simulation study of a continuous stirred tank reactor (CSTR) system to demonstrate the accuracy and

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effectiveness of the multi-model LPV model with asymmetric Gaussian weighting.

2. Description of LPV Model

For a multi-input single-output LPV system, let m inputs be $\{u_1(t), \ldots, u_m(t)\}$ at time t and the output be $y(t)$. One type of the input–output LPV system can be described as [1]

$$y(t) = \sum_{i=1}^{m} G_i(q,w)u_i(t) + v(t) \tag{1}$$

where

$$G_i(q,w) = \frac{B_i(q,w)}{A(q,w)} = \frac{\left[b_1^i(w)q^{-1} + \cdots + b_n^i(w)q^{-n}\right]q^{-d_i}}{1 + a_1(w)q^{-1} + \cdots + a_n(w)q^{-n}} \tag{2}$$

is a LPV transfer function from $u_i(t)$ to $y(t)$ that is stable, $B_i(q, w)$ and $A(q, w)$ are polynomials of q^{-1}, which denotes unit delay operator, d_i is the delay from the ith input to the output, $v(t)$ is a stationary stochastic process with zero mean and bounded variance, n is the order of the model, and $w(t)$ is the scheduling variable, which is measurable or can be calculated from other measurable process variables. In this paper, we assume that $w(t) \in [w_{\min}, w_{\max}]$, where $w_{\min}$ and $w_{\max}$ are the low and high limits of $w(t)$, respectively.

Polynomial method is a commonly used parameterization method to represent the LPV model, with parameters $b_j^i(w)$ and $a_j(w)$ replaced by polynomial functions of $w(t)$.

$$\begin{aligned} b_j^i(w) &= \gamma_1^{i,j} + \gamma_2^{i,j}w + \cdots + \gamma_{n_\beta}^{i,j}w^{n_\beta-1}, \quad j = 1,\ldots,n \\ a_j(w) &= \gamma_1^{j} + \gamma_2^{j}w + \cdots + \gamma_{n_\alpha}^{j}w^{n_\alpha-1}, \quad j = 1,\ldots,n \end{aligned} \tag{3}$$

where n_α and n_β are the orders of polynomial functions.

Eqs. (1)–(3) formulate the common approach of current LPV methods, called parameter-interpolation input–output LPV model [1]. However, it is not easy to identify parameter-interpolation LPV structure for its complex structure and fails to obtain acceptable performance in a case study [12].

3. Multi-Model LPV Model Identification

Motivated by identification practice, Zhu and Xu [10] proposed a simpler LPV model structure, which is called multi-model LPV model by Huang *et al.* [12]. Its basic principle is to identify several local linear models at fixed operating points, and then achieve the global model by interpolation *via* certain weighting functions. It has been verified [1,11,12] that the multi-model LPV is a good approximation of real processes along its operating-trajectory and the stability of this LPV model can be guaranteed easily [12].

We choose l operating points:

$$w_{\min} \le w_1 < w_2 < \cdots < w_l \le w_{\max}. \tag{4}$$

The choice of l is a trade-off of computing cost and model accuracy.

To identify the multi-model LPV model, local linear models at each fixed operating point should be determined first. The transfer function $\hat{G}_i^k(q)$ of the kth local linear models can be expressed as

$$\hat{G}_i^k(q) = \frac{B_i^k(q)}{A_i^k(q)} = \frac{\left[b_1^i(w_k)q^{-1} + \cdots + b_n^i(w_k)q^{-n}\right]q^{-d_i(w_k)}}{1 + a_1^i(w_k)q^{-1} + \cdots + a_n^i(w_k)q^{-n}}. \tag{5}$$

The parameters to be estimated for each local model can be written as

$$\hat{\theta}_k^i = \left[a_1^i(w_k)\cdots a_n^i(w_k)\; b_1^i(w_k)\cdots b_n^i(w_k)\; d_i(w_k)\right]_{(n+n+1)\times 1}. \tag{6}$$

Parameters to be estimated for all l local linear models can be denoted as

$$\Theta_L = \left[\hat{\theta}_1^1 \cdots \hat{\theta}_1^m\; \hat{\theta}_2^1 \cdots \hat{\theta}_2^m \cdots \hat{\theta}_l^1 \cdots \hat{\theta}_l^m\right]. \tag{7}$$

The values of all the parameters in Θ_L can be obtained with linear identifications using the data collected at each operating point test. Several linear identification methods can be used: prediction error method, subspace method and asymptotic method (ASYM) [15]. In an operating point test, the scheduling variable is kept constant, while a normal identification test is performed for local linear model identification using small test signals.

The multi-model global LPV model is obtained by interpolating local linear models, which can be expressed as

$$\hat{y}(t) = \sum_{k=1}^{l} \eta_k(w(t)) \sum_{i=1}^{m} \hat{G}_i^k(q)u_i(t) + v(t) \tag{8}$$

where $\hat{G}_i^k(q), i = 1, 2, \ldots, m$ are the local linear models at the kth operating point, $\eta_k(w(t)), k = 1, 2, \ldots, l$ are the weighting functions of corresponding local linear models, which are essentially static functions of scheduling variable $w(t)$, and $v(t)$ is the white noise defined in Eq. (1).

To combine all the local linear models into a global multi-model LPV model in Eq. (8), proper weighting functions are required for interpolation, which have a large effect on the accuracy of the global model. Some common weighting functions are available in literature, such as linear weight function [10] and Gaussian weight function [12]. The structures of weighting functions will be specified later.

3.1. Multi-model LPV model with linear weights

The linear weight function is the simplest one that can be preassigned. The weighting equals to the distance between current scheduling variable and operating points of the local linear models. With $\hat{y}(t|w(t))$ to be estimated where $w_k < w(t) < w_{k+1}$, $(k = 1, 2, \ldots, l-1)$, the weighted output $\hat{y}(t)$ is

$$\hat{y}(t) = \frac{w_{k+1} - w}{w_{k+1} - w_k} \sum_{i=1}^{m} \hat{G}_i^k(q)u_i(t) + \frac{w - w_k}{w_{k+1} - w_k} \sum_{i=1}^{m} \hat{G}_i^{k+1}(q)u_i(t). \tag{9}$$

Although linear weighting can be used conveniently in the multi-model LPV model, it is not accurate enough to capture the full dynamic behavior of nonlinear process.

3.2. Multi-model LPV model with Gaussian weights

A preferable choice for determining model weights is Gaussian function, which can be written as

$$\eta_k(w(t)) = \frac{\hat{\alpha}_k(w(t))}{\sum_{j=1}^{l} \hat{\alpha}_j(w(t))}, k = 1, 2, \ldots, l \tag{10}$$

where

$$\hat{\alpha}_k(w(t)) = \exp\left[-\frac{1}{2}\left(\frac{w(t) - w_k}{\sigma_k}\right)^2\right] \tag{11}$$

and σ_k represents the width coefficient of the kth local linear model.

In the realm of multi-model structures [13] and fuzzy sets [14], Gaussian weighting functions have been widely utilized, which have