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On the experimental determination of growth and damping rates for combustion instabilities

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ABSTRACT

This paper presents four experimental methods for the evaluation of growth rates of combustion instabilities. A systematic investigation is conducted on a laminar slot burner with five operating points (two stable and three unstable). The accuracy of the methods is assessed by cross comparison and the use of three different flow variables as input: velocity, pressure and heat release rate fluctuations. Finally, the experimental determinations of the growth rates are compared to the prediction of a low-order acoustic model fed with a Flame Transfer Function.

1. Introduction

Combustion instabilities (CI) are a major problem for the design and operation of power-generation systems such as gas turbines, aeronautical engines and rocket engines [1]. The constructive coupling between acoustic waves and unsteady heat release rate constitutive of CI, is responsible for loss of performance, restriction under operating conditions and sometimes catastrophic failures. The challenge for understanding and predicting CI lies in the multiplicity of physical phenomena involved in the unstable loop [2]: acoustics, vortex dynamics, mixing, turbulence, chemistry, two-phase flows, etc. One of the canonical configurations for the study of CI is the laminar premixed flame, which has been extensively scrutinized [3–6]. Despite its apparent simplicity, this flame contains one of the main physical mechanisms driving CI: the dynamic interaction between acoustics, vortical structures in the flow field and the flame front. Comparative studies of conical, V-shaped and triangular flames gives insight into the mechanisms driving the overall response of the flame to acoustic perturbations [7]. The crux of the matter for the prediction of CI is to model this response, which is often achieved through a so-called Flame Transfer Function (FTF), relating the unsteady heat release rate of the flame, $\dot{q}'$, to the incident acoustic velocity perturbation, v'. In many of the above mentioned configurations, reasonably accurate analytical models for the FTF can be derived [6,8]. For realistic combustion devices, the FTF must be measured experimentally [9–11] or computed with high-fidelity unsteady 3D numerical simulations [12–14].

For the design of stable combustion devices, a trial-and-error experimental approach is very costly so that alternatives using numerical simulation are sought. The first option, which may be called a brute-force approach is to solve the reacting Navier–Stokes equations over the whole configuration. Large-Eddy Simulation, for example, has shown its potential for this task [15,16]. The computational cost of a single simulation is however so large that the exploration of the whole range of operating conditions or design variations is impractical. An alternative is to solve only for acoustic perturbations and model the flame via the FTF: the resulting tool may be called a thermoacoustic code. Whether the equations be Linearized Euler or Helmholtz, in complex 3D geometries or 1D network models, the methodology is usually computationally lean and has shown its ability to predict stability maps [10,17–20]. The typical output of a thermoacoustic solver is threefold: (1) the frequency of the instability, (2) the shape of the associated pressure and velocity fields and (3) the linear growth rate of individual eigenmodes.

Because the impact of the flame as an active acoustic source is usually small, the prediction of eigenfrequencies and mode shapes is not a difficult task. The determination of the growth rates is more subtle as it requires a precise evaluation of the flame response (with a FTF for example), the fluxes through the boundaries (by specifying the impedances) and the sources of internal dissipation (through friction at the walls and transfer from acoustics to vorticity). And yet, a precise determination of the linear growth rate is crucial for the design and safe operation of an engine. Indeed, even if a given operating point is observed as stable, it is

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Nomenclature

Greek symbols

α	Natural damping of the system under non-reacting conditions [s $^{-1}$]
β	Growth/damping rate induced by the flame [s $^{-1}$]
δ	Impulse signal in the IR test [Volts]
η	Any state variable
κ	Coefficient that accounts for the non-linearity of the flame response
ν	Growth/damping rate of the system [s $^{-1}$]
ω	Angular frequency [Rad/s]
ω_0	Response/resonance angular frequency [Rad/s]
$\omega_{0,\,nr}$	Response angular frequency under non-reacting configuration [Rad/s]
Φ	Equivalence ratio [-]
ϕ	Phase [Rad]
φ	Phase of the FTF [Rad]
τ	Time delay between the velocity and heat release rate signals [s]
ξ	Forcing term
ζ	Uncorrelated with noise

Roman letters

A	Amplitude of the limit cycle
a	Acceleration [G]
A_0	Amplitude of the limit cycle without stochastic forcing
A_{IR}	Amplitude of the impulse signal in the IR test [Volts]
$\mathcal{C}$	Combustion noise factor [-]
c_0	Normalized stochastic forcing level [-]
E	Fresh to burnt-gases density ratio [-]
$\mathcal{F}$	Flame Transfer Function [-]
f	Frequency [Hz]
f_0	Response frequency [Hz]
$f_{0,\,nr}$	Response frequency under no-reacting configuration [Hz]
$\mathcal{G}$	Gain of the FTF [-]
h	Effective slot height [m]
h_s	Slot height [mm]
I	Intensity of CH* radical [Volts]
l_s	Slot length [mm]
$\mathcal{N}$	Combustion interaction index [-]
p	Acoustic pressure [Pa]
$\dot{q}$	Heat release rate [$\propto$Js $^{-1}$]
r	Pinching distance [m]
$\mathcal{S}$	Power spectral density
S_s	Slot cross section [m^2]
t	Time [s]
T_g	Fresh gases temperature [°C]
T_s	Slot temperature [°C]
v	velocity [m/s]
v_b	Bulk velocity [ms $^{-1}$]
w_s	Slot width [mm]

useful to know how far from stability it is so that small changes (manufacturing variations, aging through mechanical and thermal stress, variations in fuel properties, etc.) do not trigger a CI.

The objective of the present work is to present various experimental techniques that can be used to measure the growth rate of stable or unstable modes with the intent to serve as a validation for thermoacoustic solvers. The manuscript is organized as follows: First, in Section 2 a general model for a second-order dynamic system is presented. This model is the starting point to derive the particular solutions to the corresponding four damping/growth rate identification methods. Then, the experimental setup and diagnostics are presented in Section 3. The experimental results for each of the four identification methods is then discussed in Section 4. Finally, a low-order model is derived in Section 5 and compared to the results of Section 4 for the validation of its ability to predict the linear growth rate of CIs.

2. Theoretical model

The objective of this work is the evaluation of the growth rate for a specific mode and as explained in [19,21], around a given eigenmode of angular frequency ω_0, it is a valid assumption to consider that the thermoacoustic system obeys a second-order differential equation of the form:

$$\ddot{\eta} - f(\eta, \dot{\eta}) + \omega_0^2 \eta = \xi \tag{1}$$

This formulations follows that of [21], where η represents any state variable and f is a non-linear function of η and its first derivative $\dot{\eta}$, which accounts for internal dissipation, fluxes of acoustic energy at the boundaries and the contribution of unsteady combustion, i.e. the flame. ξ, corresponds to a forcing term due to an external device such as a loudspeaker. In the present study, η can be the acoustic pressure p' or acoustic velocity v' at a given location in the experimental rig, or the heat release rate integrated over the entire combustion chamber, $\dot{q}'$. Following [21], the function f may be modeled as:

$$f(\eta, \dot{\eta}) = \dot{\eta}(\beta - \alpha - \kappa \eta^2) \tag{2}$$

where α corresponds to the contribution of linear acoustic absorption in the volume and fluxes at the boundaries, meaning that it is positive. β is the linear contribution of the feedback induced by the flame. It is positive when the flame drives the instability and negative when unsteady combustion damps acoustic fluctuations. The coefficient κ accounts for the non-linearity of flame response to acoustics and controls the amplitude of the limit cycle reached when the system is linearly unstable. The growth/damping rate, ν, of the system is defined as:

$$\nu = \frac{\beta - \alpha}{2}. \tag{3}$$

Without combustion, $\beta = 0$, then $\nu = -\alpha/2$, so that the system is always stable. In a reacting flow, if $\beta > \alpha$ then $\nu > 0$, meaning that the driving contribution of the flame overcomes the linear acoustic losses and the system becomes linearly unstable. On the other hand, if $\beta < \alpha$ the system is linearly stable.

Eqs. (1)–3 provide the proper background for the extraction of the deterministic quantities ω_0 and ν from experimental data. Four identifications methods are now presented: Harmonic Response (HR), Impulse Response (IR), Active Control (AC) and White Noise (WN).

2.1. Harmonic response method (HR)

The harmonic response has been extensively used in the study of combustion instabilities for non-reacting [22] and reacting [20,23] flows. However, this technique is limited to systems featuring negative growth rates, i.e. $\nu < 0$. For a linearly stable system, the non-linear part of Eq. (2) can be neglected as long as perturbations to the equilibrium state are small so that Eq. (1) reduces to:

$$\ddot{\eta} - 2\nu\dot{\eta} + \omega_0^2 \eta = \xi \tag{4}$$

Taking the power spectral density of Eq. (4) yields:

$$\mathcal{S}_\eta = \frac{\mathcal{S}_\xi}{(\omega_0^2 - \omega^2)^2 + 4\nu^2\omega^2} \tag{5}$$