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Robustness of reinforced gradient-type iterative learning control for batch processes with Gaussian noise☆

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ABSTRACT

In this paper, a reinforced gradient-type iterative learning control profile is proposed by making use of system matrices and a proper learning step to improve the tracking performance of batch processes disturbed by external Gaussian white noise. The robustness is analyzed and the range of the step is specified by means of statistical technique and matrix theory. Compared with the conventional one, the proposed algorithm is more efficient to resist external noise. Numerical simulations of an injection molding process illustrate that the proposed scheme is feasible and effective.

1. Introduction

In industrial community, petrochemical processes, microelectronic manufacturing and metallurgical processes are typical batch processes, each of which repetitively executes a given task over a fixed duration [1]. For conventional practical execution, an industrial batch process is tuned by a proportional-integral-derivative (PID) controller so that the controlled system may operate with desired performance [2,3]. However, for some plants, the PID-controller-tuned systems operate with unsatisfactory transient performance such as slower response with longer settling time or faster response with oscillatory overshoot [4,5]. As a consequence, the dissatisfaction may influence the product quality when the controlled system operates repetitively. Under this circumstance, it is necessary to design a PID controller in a trade-off mode. To improve the transient performance, some intelligent techniques have been developed such as expertise knowledge for selecting piecewise PID controller gains [6,7] and iterative learning control (ILC) strategies for generating a sequence of upgraded control commands [8–13] and so on.

The ILC strategy was first proposed by Arimoto *et al.*, with an ILC scheme applied to a robotic manipulator while it is repetitively attempting to track a desired trajectory [12]. The basic mechanism of the ILC strategy is to generate the control signal for the next operation by compensating for the current control signal with its proportional tracking error, error derivative and/or error integral so that the tracking performance of the next operation gets better. Owing to its good learning efficiency and a prior less system knowledge requirement, the ILC theme has attracted much attention not only in robotic society for manipulator's trajectory tracking but also in industrial fields such as batch process for transient performance improvement [8–13] and CD disk recording [14,15]. The fundamental ILC updating rules are constructed on the basic postulate that the desired trajectory is iteratively invariant while the system repetitively operates over a fixed finite time interval with resettable initial states [16,17]. One of the key ILC contributions is convergence analysis, which has been progressive by assessing the tracking error in forms of lambda-norm, Lebesgue-p norm or discrete frequency Parseval energy [12,18,19] and so on. For practical applicability, the robustness of ILC schemes to system parameter uncertainty, noise perturbation and initial state shifts has been analyzed [20–22]. Besides, when the system dynamics is identified, the system information is utilized to construct optimized/optimal ILCs for fastening the learning convergence [23–29].

For the optimized ILCs, the main efforts have been made on the norm-optimal ILC and the parameter-optimized ILC [23–26]. In these investigations, Both ILCs can guarantee the tracking error measured for mean-square norm to monotonically reduce, though its robustness to external noise is not involved. In addition, as typical optimization methodologies, Newton and quasi-Newton methods have been harnessed to compose optimized iterative learning control updating laws [27,28], where the convergence is analyzed regardless of the robustness. Progressively, a gradient-type iterative learning control (GILC) updating law has been constructed for the system with uncertainty [29], where the weighted learning gain consists of a scalar matrix and the robust monotone convergence of the GILC scheme is achieved. However, its learning performance gives milder convergence and

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weaker resistance to uncertain noise. The reason is perhaps that the scalar learning gain matrix does not sufficiently take the advantage of system knowledge. Besides, because the searching path of the gradient-type ILC algorithm is saw-tooth and the learning step gets very small when the output closes the desired trajectory, especially, when the system is ill-conditioned, the tracking behavior of the GILC scheme goes tagging. Thus the scheme needs to be reinforced with system knowledge so as to enhance the tracking performance. Additionally, as system external noise is inevitable in practical application, the robustness of the learning scheme to noises is necessarily explored.

This paper develops a reinforced gradient-type iterative learning control (RGILC) algorithm for a class of discrete linear time-invariant systems with external Gaussian noise. The idea is to make use of system matrices and a proper learning step to weigh the gradient. The robustness of the RGILC algorithm to external noise is analyzed in virtue of mathematical expectation and the range of learning step is specified for convergent assumption. Numerical simulations are presented to illustrate the validity as well as the effectiveness.

2. RGILC Algorithm and Preliminaries

Consider a class of discrete linear time-invariant single-input, single-output batch process control systems as follows.

$$\begin{cases} \boldsymbol{x}_k(i+1) = \boldsymbol{A}\boldsymbol{x}_k(i) + \boldsymbol{B}(u_k(i) + \boldsymbol{\xi}_k(i)), \\ y_k(i+1) = \boldsymbol{C}\boldsymbol{x}_k(i+1) + \varsigma_k(i+1), \\ \boldsymbol{x}_k(0) = 0, \quad i \in S \end{cases} \tag{1}$$

where $S = \{0, 1, 2, \dots, N\text{-}1\}$ denotes the set of discrete time sampling with N referring to the total sampling numbers, index i stands for the sampling number and subscript $k \in \mathbb{N}$ marks the iteration or batch index, $\boldsymbol{x}_k(i) \in \mathbb{R}^n$, $u_k(i) \in \mathbb{R}$ and $y_k(i) \in \mathbb{R}$ are n-dimensional state, scalar input and scalar output at the ith sampling time of the kth iteration, $\boldsymbol{\xi}_k(i) \in \mathbb{R}$, $i \in S$ and $\varsigma_k(i+1) \in \mathbb{R}$, $i \in S$ are load noise and measurement noise, respectively, $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are constant system matrices with appropriate dimensions satisfying $\boldsymbol{CB} \neq 0$. When the learning process is realizable, *i.e.*, for a desired trajectory $y_d(i)$, $i \in S$, there exists a unique control input signal $u_d(i)$ such that

$$\begin{cases} \boldsymbol{x}_d(i+1) = \boldsymbol{A}\boldsymbol{x}_d(i) + \boldsymbol{B}u_d(i), \\ y_d(i+1) = \boldsymbol{C}\boldsymbol{x}_d(i+1), \ i \in S. \end{cases} \tag{2}$$

We denote

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{CB} & 0 & 0 & \cdots & 0 \\ \boldsymbol{CAB} & \boldsymbol{CB} & 0 & \cdots & 0 \\ \boldsymbol{CA}^2\boldsymbol{B} & \boldsymbol{CAB} & \boldsymbol{CB} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{CA}^{N-1}\boldsymbol{B} & \boldsymbol{CA}^{N-2}\boldsymbol{B} & \boldsymbol{CA}^{N-3}\boldsymbol{B} & \cdots & \boldsymbol{CB} \end{bmatrix}_{N\times N}, \boldsymbol{u}_k = \begin{bmatrix} u_k(0) \\ u_k(1) \\ \vdots \\ u_k(N-1) \end{bmatrix},$$

$$\boldsymbol{y}_k = \begin{bmatrix} y_k(1) \\ y_k(2) \\ \vdots \\ y_k(N) \end{bmatrix}, \boldsymbol{\xi}_k = \begin{bmatrix} \xi_k(0) \\ \xi_k(1) \\ \vdots \\ \xi_k(N-1) \end{bmatrix} \text{ and } \boldsymbol{\varsigma}_k = \begin{bmatrix} \varsigma_k(1) \\ \varsigma_k(2) \\ \vdots \\ \varsigma_k(N) \end{bmatrix},$$

where, $\boldsymbol{H}$ is termed as Markov parameter matrix of system (1), vectors $\boldsymbol{u}_k$, $\boldsymbol{y}_k$, $\boldsymbol{\xi}_k$ and $\boldsymbol{\varsigma}_k$ are named as input, output, load noise and measurement noise super vectors, respectively. Then system (1) is compacted as

$$\boldsymbol{y}_k = \boldsymbol{H}(\boldsymbol{u}_k + \boldsymbol{\xi}_k) + \boldsymbol{\varsigma}_k. \tag{3}$$

Let $\boldsymbol{y}_d = [y_d(1), y_d(2), \dots, y_d(N)]^T$ be a predetermined desired trajectory that the system output should follow as an ideal target, where T denotes the transpose operator and $\boldsymbol{e}_k = \boldsymbol{y}_d - \boldsymbol{y}_k$ denotes the tracking error vector. The objective of developing an iterative learning control algorithm for system (3) is to generate a sequence of input super vectors $\{\boldsymbol{u}_k\}$ so that it may stimulate the system (3) to track the desired trajectory $\boldsymbol{y}_d$ as precisely as possible as the iteration index goes to infinity, namely,

$$\lim_{k\to\infty} \mathrm{E}\left\{\|\boldsymbol{e}_k\|_2^2\right\} \le \varepsilon \tag{4}$$

where $\|\cdot\|_2$ denotes 2-norm of a vector and $\mathrm{E}\{\cdot\}$ represents the mathematical expectation operator.

Before mentioning the ILC scheme, it is worthy to mind that, theoretically, the desired control input vector can be achieved by inversing the system as $\boldsymbol{u}_d = \boldsymbol{H}^{-1}\boldsymbol{y}_d$ regardless of noise when the Markov matrix $\boldsymbol{H}$ is invertible. However, in reality, especially for rapid responding dynamics with large-scale Markov matrix, the inversion technique needs a complex computation, which is sensitive to system parameter perturbation or computation error accumulation. Sometimes, the inversion method incurs divergence of the learning scheme [8].

One of feasible ILC manners to make use of system knowledge is a gradient-type ILC updating mechanism briefed as follows.

For system (3), define a sequence of iteration-wise quadratic objective functions taking forms of

$$\min_{\boldsymbol{u}_k} J(\boldsymbol{u}_k) = \frac{1}{2}\|\boldsymbol{y}_d - \boldsymbol{H}\boldsymbol{u}_k\|_2^2. \tag{5}$$

It is easy to derive that the gradient of function $J(\boldsymbol{u}_k)$ with respect to argument $\boldsymbol{u}_k$ is $\nabla J(\boldsymbol{u}_k) = -\boldsymbol{H}^T\boldsymbol{e}_k$. Then a (descent) gradient-type ILC (GILC) scheme is constructed as

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + \boldsymbol{\Lambda}\boldsymbol{H}^T\boldsymbol{e}_k \tag{6}$$

where $\boldsymbol{\Lambda} = \alpha\boldsymbol{I}$ represents a scalar learning gain matrix with learning step α. The details may refer to Ref. [29].

For the GILC algorithm (6), by replacing its learning gain matrix $\boldsymbol{\Lambda}$ with a symmetric matrix $\alpha(2\boldsymbol{I} - \alpha\boldsymbol{H}^T\boldsymbol{H})$, a reinforced GILC (RGILC) updating law is developed as follows: $\boldsymbol{u}_1$: given arbitrarily;

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + \alpha\left(2\boldsymbol{I} - \alpha\boldsymbol{H}^T\boldsymbol{H}\right)\boldsymbol{H}^T\boldsymbol{e}_k, k = 1, 2, \cdots. \tag{7}$$

2.1. Basic Assumptions

(A1) Load noise super vectors $\boldsymbol{\xi}_l$, $l \in \mathbb{N}_+$ and measurement noise super vectors $\boldsymbol{\varsigma}_h$, $h \in \mathbb{N}_+$ are Gaussian white-type. Mathematically, the covariance matrices with respect to $\boldsymbol{\xi}_l$ and $\boldsymbol{\varsigma}_l$ satisfy the following relations

$$\begin{cases} \mathrm{E}\left\{\boldsymbol{\xi}_k\boldsymbol{\xi}_k^T\right\} = \overline{\sigma}^2\boldsymbol{I}, \\ \mathrm{E}\left\{\boldsymbol{\xi}_k\boldsymbol{\xi}_l^T\right\} = \boldsymbol{0}, \quad (l \neq k) \end{cases}, \quad \begin{cases} \mathrm{E}\left\{\boldsymbol{\varsigma}_k\boldsymbol{\varsigma}_k^T\right\} = \hat{\sigma}^2\boldsymbol{I}, \\ \mathrm{E}\left\{\boldsymbol{\varsigma}_k\boldsymbol{\varsigma}_h^T\right\} = \boldsymbol{0}, \quad (h \neq k) \end{cases}$$

where $\overline{\sigma}^2$ denotes the variance of load noise $\boldsymbol{\xi}_{(i)}$, $i = 0,1,2,\dots,N$, $l \in \mathbb{N}_+$, $\hat{\sigma}^2$ refers to the variance of measurement noise $\boldsymbol{\varsigma}_{h,l}(i)$, $i = 0,1,2,\dots,N$, $h \in \mathbb{N}_+$, $\boldsymbol{0}$ stands for a zero matrix, and $\boldsymbol{I}$ represents an identity matrix with appropriate dimensions.

(A2) Load noise super vectors $\boldsymbol{\xi}_l$, $l \in \mathbb{N}_+$ and measurement noise super vectors $\boldsymbol{\varsigma}_h$, $h \in \mathbb{N}_+$ are independent, that is, $\mathrm{E}\{\boldsymbol{\xi}_l\boldsymbol{\varsigma}_h^T\} = \boldsymbol{0}$ and $\mathrm{E}\{\boldsymbol{\varsigma}_l\boldsymbol{\xi}_h^T\} = \boldsymbol{0}$ hold for any $l, h \in \mathbb{N}_+$.

Assume that $\boldsymbol{Q} = (q_{hl})_{n\times n}$ is a real square symmetric matrix. The eigenvalues set of $\boldsymbol{Q}$ are denoted as $\{\lambda_j(\boldsymbol{Q})\}_{j=1}^n$, while its maximal and the minimal eigenvalues are denoted as $\lambda_{\max}(\boldsymbol{Q}) = \max_{1\le j\le n}\{\lambda_j(\boldsymbol{Q})\}$ and $\lambda_{\min}(\boldsymbol{Q}) = \min_{1\le j\le n}\{\lambda_j(\boldsymbol{Q})\}$, respectively. The spectral radial is defined as $\rho(\boldsymbol{Q}) = \max_{1\le j\le n}\{|\lambda_j(\boldsymbol{Q})|\}$ and the singular values of $\boldsymbol{Q}$ are defined as $\{\sigma_j(\boldsymbol{Q})\}_{j=1}^n = \left\{\sqrt{\lambda_j(\boldsymbol{Q}^T\boldsymbol{Q})}\right\}_{j=1}^n$. The trace of matrix