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Theoretical analysis of the mass balance equation through a flame at zero and non-zero Mach numbers

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ABSTRACT

This paper discusses a classical paradox in thermoacoustics when jump conditions are derived for acoustic waves propagating through a thin flat flame. It shows why volume conservation must be used for perturbations at zero Mach number (continuity of $v' = u'A$) while mass conservation is used at non-zero Mach numbers (continuity of $m' = \bar{\rho}_0 u'A + \bar{u}_0 \rho' A$). First, from the three-dimensional mass balance equation, a quasi one-dimensional mass balance equation is obtained for surface-averaged quantities. Then it is demonstrated that the acoustic and entropy disturbances are coupled and need to be solved together at the flame front because singularities in the entropy profile affect mass conservation. At non-zero Mach number, the entropy generated in the thin flame is convected by the mean flow: no singularity occurs and leads to the classical mass conservation at the interface. However, at zero Mach number, the flow is frozen and entropy spots are not convected downstream: they produce a singularity at the flame front due to the mean density gradient, which acts as an additional source term in the mass conservation equation. The proper integration of this source term at zero Mach number leads, not to the mass, but to the volume flow rate conservation of perturbations. A balance equation for the volume flow rate has been also derived. This equation couples the volume flow rate and the mean and fluctuating pressure. This latter equation degenerates naturally toward the volume flow rate conservation at the flame interface at zero Mach number because of the pressure continuity. This theoretical analysis has been compared to LEE (Linearized Euler Equation) simulations of stable flames and a good agreement is found for the entropy fluctuations shape and the conserved quantities.

1. Introduction

Acoustics remains a crucial topic in the development of modern gas turbines: acoustic waves can propagate in the whole combustion chamber, interacting with the compressor exit, the turbine stator inlet or the flames, leading to the production of direct [1–3] and indirect noise [4–8], vibrations and combustion instabilities [9–13].

Describing the acoustic modes, which can appear in combustion chambers and finding methods to control them has been the topic of multiple studies over the last decades [9,11,12,14–20]. The complexity and the cost of performing laboratory-scale experiments explain why progress in this field has been slow for a long time since. Recently, new well-instrumented acoustic experiments [7,14,21,22] have opened the path to investigate flame response to acoustics [23], direct and indirect noise [7] as well as combustion instabilities [10,14,15,21,22]. In addition, theoretical and numerical approaches have progressed in different directions: (1) three-dimensional high fidelity simulations of combustion chambers have been performed [24–27], (2) three-dimensional acoustic tools have been developed [28–31] and (3) analytical approaches have been proposed to describe acoustics in simplified configurations at low cost [4,5,8,16,32–35]. In particular, this last approach allows the investigation of the underlying mechanisms involved in acoustic phenomena since explicit expressions of acoustic sources or growth rates of modes are obtained.

These low-order methods for thermoacoustics are usually based on a one-dimensional formalism in which acoustic waves are propagated in a network. A paradox arises from the fact that acoustic modeling is usually performed at zero Mach number ($\bar{u}_0 = 0$) while

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combustion is a process necessarily place at non zero Mach number (otherwise reactants are frozen and are never transported to the reaction zone leading to zero mean heat release $\overline{Q}_0 = 0$, i.e. no temperature or density gradients). A common approach is therefore to consider two "worlds": the first one is the "acoustic world" at zero Mach number and the second one is a "convective world" required by the flame to create the density/temperature gradients at the flame front. Flame Transfer Functions used in Helmholtz solvers are a typical example of how a convective quantity – the time-delay – is incorporated into the "acoustic world" which assumes a zero Mach number. Low-order models are usually prone to this paradox when dealing with acoustic jump conditions required to link fluctuating acoustic quantities at both sides of a thin flame: for a thin flame located at a section change (Fig. 1) in the limit of zero Mach number, typical thermoacoustics studies [11,17,21,30,33,35–40] incorporate a jump condition corresponding to the continuity of the volume flow rate to express velocity perturbation u'_1 and u'_2 on both sides of the flame:

$$u'_1 A_1 = u'_2 A_2 \tag{1}$$

while the intuitive condition would be to write mass conservation:

$$\bar{\rho}_1 u'_1 A_1 = \bar{\rho}_2 u'_2 A_2 \tag{2}$$

which includes the mean density values on both sides of the flame and differs strongly from Eq. (1). Mass conservation is actually used at non-zero Mach numbers by some authors [18,36,41–43], leading to some confusion in the community. The question becomes more complex in network models where Mach number can be zero in certain parts of the combustion chamber modeled as one-dimensional tubes and non null in others (Fig. 1). A crucial question is therefore to prove the consistency between jump conditions at non-null Mach number ($M \neq 0$) and the limit case when the Mach number goes to zero ($M \to 0$). Moreover, the differences between Eqs. (1) and (2) are large because the ratio $\bar{\rho}_1/\bar{\rho}_2$ is of the order of 5–10 in most flames. Using Eq. (1) or (2) leads to very different results in Helmholtz solvers. Therefore, understanding which velocity jump condition must be used is a critical building block in all Helmholtz formulations which clearly requires a careful analysis.

The present paper tries to elucidate this paradox by deriving jump conditions for mass and volume flow rates on a thin flame front at zero and non-zero Mach number. The first starting point is to write the mass conservation at non-null Mach number (Section 2.1). This balance equation is valid but does not degenerate simply to the proper equation at zero Mach number where the volume flow rate is conserved and not the mass flow rate [11]. Another starting point is to write the conservation of total enthalpy at the interface (Section 2.2), which leads to volume flow rate conservation (Eq. (1)) for zero Mach numbers. Showing why these approaches are actually compatible is one goal of the present paper. To achieve this, jump conditions for both mass ($\hat{m} = \bar{\rho}_0 \hat{u} A + \hat{\rho} \bar{u}_0 A$) and volume ($\hat{v} = \hat{u} A$) flow rate perturbations are derived in a case corresponding to two tubes connected by a passive flame and section change (Fig. 1). From the three-dimensional mass balance equation, a quasi-one dimensional mass balance equation is obtained for surface-averaged quantities in Section 2. Then the mass flow rate conservation equation is derived in Section 2.1 for all Mach numbers. This equation couples the unsteady mass flow rate $\hat{m}$ and the entropy fluctuations $\hat{s}$. In addition, a conservation equation for the volume flow rate is also obtained in Section 2.2, which couples the unsteady flow rate $\hat{v}$ and the fluctuating pressure $\hat{p}$. The comparison of the mass and volume flow rate equations in Section 2.3 shows that entropy $\hat{s}$ and pressure gradient $\frac{d\bar{p}}{dx}$ singularities present in these equations change with the Mach number and explains why mass flow rate is conserved at non-null Mach numbers (Section 3) and volume flow rate at zero Mach number (Section 4) demonstrating the consistency between the two formulations.

2. Mass and volume flow rate formulation

The conservation of the fluctuating mass and volume flow rate through the thin flame front of Fig. 1 is described for a configuration with a "steady" flame, i.e. no heat release fluctuations ($\widehat{Q} = 0$) and $\rho_1 > \rho_2$ due to a different temperature in the fresh mixture (subscript 1) and the hot mixture (subscript 2). No distinction between null or non-null Mach number is necessary at this step.

2.1. Mass flow rate ($\hat{m}$)/entropy ($\hat{s}$) coupled equations

The local mass conservation reads:

$$\frac{\partial \rho}{\partial t} = -div(\rho u) \tag{3}$$

where ρ and u are instantaneous three-dimensional quantities.

Since the case studied is quasi-one-dimensional, a spatial averaging over the area A is applied:

$$\overline{F} = \frac{1}{\bar{\rho} A} \int_A \rho F dA \tag{4}$$

where F corresponds to any quantity such as pressure and velocity and $\bar{\rho} = \frac{1}{A} \int_A \rho dA$.

Eqs. (3) and (4) lead to a one-dimensional mass balance equation:

$$A \frac{\partial \bar{\rho}}{\partial t} = -\frac{\partial}{\partial x} (\bar{\rho} \bar{u} A) \tag{5}$$

This equation can be linearized around the mean state:

$$A \frac{\partial \rho'}{\partial t} = -\frac{\partial}{\partial x} (\rho' \bar{u}_0 A + \bar{\rho}_0 u' A) \tag{6}$$

where any one-dimensional quantity $\overline{F}$ is decomposed as $\overline{F} = \overline{F}_0 + F'$ where $\overline{F}_0$ is the mean quantity and F' is the fluctuating part. The second-order term $\rho' u' A$ has been neglected.

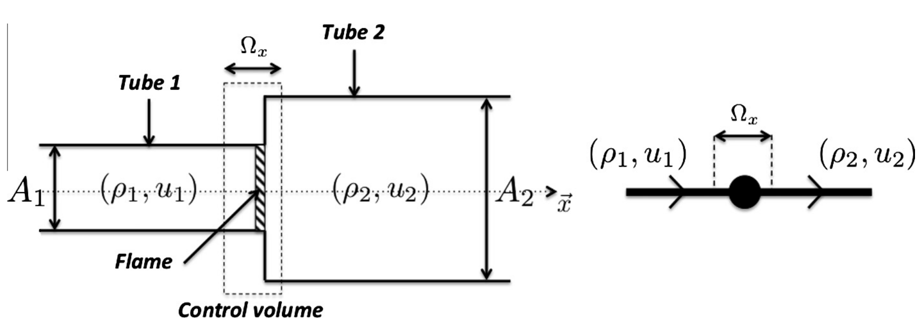

Fig. 1. Configuration (left) with the corresponding one-dimensional model (right) and the control volume (- - -): two tubes connected by a flame and an abrupt change of section from A_1 to A_2.