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Multi-objective nonlinear model predictive control through switching cost functions and its applications to chemical processes☆

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ABSTRACT

This paper proposes a switching multi-objective model predictive control (MOMPC) algorithm for constrained nonlinear continuous-time process systems. Different cost functions to be minimized in MPC are switched to satisfy different performance criteria imposed at different sampling times. In order to ensure recursive feasibility of the switching MOMPC and stability of the resulted closed-loop system, the dual-mode control method is used to design the switching MOMPC controller. In this method, a local control law with some free-parameters is constructed using the control Lyapunov function technique to enlarge the terminal state set of MOMPC. The correction term is computed if the states are out of the terminal set and the free-parameters of the local control law are computed if the states are in the terminal set. The recursive feasibility of the MOMPC and stability of the resulted closed-loop system are established in the presence of constraints and arbitrary switches between cost functions. Finally, implementation of the switching MOMPC controller is demonstrated with a chemical process example for the continuous stirred tank reactor.

1. Introduction

Nonlinear model predictive control (NMPC) has been receiving great attention since the 1990s in both industries and academia [1–4], especially in chemical processes [5–9]. In general, NMPC uses a model of plants to predict their future evolutions and a control input is determined by online optimization to minimize a certain performance criterion subject to the state and/or the control constraints. The input obtained is applied until the next sampling time and the overall procedure is repeated. Clearly, the cost functions that represent the performance criteria of controllers play an important role in the design of NMPC controllers.

In many MPC applications to chemical processes [5–9], not only one but a number of different performance criteria should be taken into account for the controlled system design. At the same time, chemical processes are usually characterized by nonlinear behavior and strong coupling of various process variables. These features lead to a multi-objective NMPC problem. Different performance criteria to be considered may be associated with different regions in the system state space or with different time instants or stages in processes. Take the grade transition processes of polyolefin plants [8,9] as an example. The prices of produced polymer resins and consumed energy change according to certain schedules dependent on market conditions. Consequently, we should use several different cost criteria to design different controllers for the grade transition process in order to meet the economic optimization for the polyolefin plant. It is also necessary to quickly react to disturbances or faults whenever they occur [10].

To tackle the problem, several new multi-objective NMPC algorithms have been proposed recently. For instance, the MPC control sequence is computed to minimize the max of a finite number of objectives [11]. The linear MPC law is derived by minimizing a convex combination of different cost functions and stability is guaranteed by the convex combination, which is close to that desired [12]. The lexicographic programming and logic method are used to prioritize multiple cost functions and then design multi-objective NMPC controllers [13–15]. The MPC controller is derived by minimizing the distance of the cost function to that of the steady-state utopia point and the nominal asymptotic stability of the NMPC is ensured by the terminal equality constraint and the assumption of strong duality [16]. Similarly, we have proposed a utopia-tracking multi-objective NMPC scheme in the dual-mode framework using the terminal region constraint and control Lyapunov functions [17].

Different cost functions associated with a certain state region individually and a state-dependent switch between the cost functions have been employed to design the multi-objective NMPC controller [18]. Stability of the proposed NMPC is ensured by imposing a constraint to the optimization problem such that if a switch occurs at a certain sampling time, the optimal value of the current activated cost function must be smaller than one of the cost functions that are active at the last time. Moreover, general time-dependent switches between

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different cost functions are exploited to compute multi-objective NMPC law and the average dwell-time method is used to ensure asymptotic stability of the proposed NMPC [19]. This means that in average, switches between different cost functions do not occur very often, *i.e.*, the switches are restricted to guarantee the stability of the controller.

In this paper, we consider a class of multi-objective NMPC problems, where different cost functions to be minimized may be switched and then activated at any sampling time. This implies that at each sampling time, one of the cost functions is selected and the cost functions can be switched arbitrarily. This operation results in a closed-loop switched system since we switch different NMPC controllers designed by minimizing different cost functions. In order to guarantee recursive feasibility of the switching NMPC and stability of the closed-loop system, we use the dual-mode control method [20,21]. In this method, a local control law with some free-parameters is designed by the control Lyapunov functions (CLFs) concept [22] to enlarge the terminal state set of MPC. Thus the correction term is computed when the states are out of the terminal set and the free-parameters of the local control law are computed when the states are in the terminal set. We establish the recursive feasibility of the MPC and stability of the closed-loop system in the face of constraints and arbitrarily switched cost functions. This is a nice result since one can incorporate arbitrary change in the desired cost functions, which allows us to take into account a larger class of multi-objective control problems. The contribution of the paper is then a step forward in the design of multi-objective NMPC controllers that can take into consideration switches between different cost functions, going beyond the existing modern tools from switched system theory. Finally, the implementation of the proposed multi-objective NMPC controller is demonstrated using a chemical process example of the continuous stirred tank reactor (CSTR).

2. Problem Setup and Preliminaries

We consider a continuous-time nonlinear system described by

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{f}(\boldsymbol{x}(t))+\boldsymbol{g}(\boldsymbol{x}(t))\boldsymbol{u}(t),\quad t\geq 0,\quad \boldsymbol{x}(0)=\boldsymbol{x}_0 \tag{1}$$

where $\boldsymbol{x}(t)\in X\subset R^n$ is the state at time t, $\boldsymbol{u}(t)\in X\subset R^m$ is the control input at time t and state transition mappings $\boldsymbol{f}$: $X\rightarrow R^n$ and $\boldsymbol{g}$: $X\rightarrow R^{n\times m}$ with $\boldsymbol{f}(0)=0$ and $\boldsymbol{g}(0)=0$. Without loss of generality, we assume that the origin is an equilibrium point of the null-input system. The states and controls are required to satisfy constant pointwise-in-time constraints

$$(\boldsymbol{x}(t),\boldsymbol{u}(t))\in Z,\quad t\geq 0 \tag{2}$$

for some compact set $Z\subseteq X\times U$. Set Z is assumed to contain the equilibrium point as its interior.

Let $R_{\geq 0}$ denote the set of non-negative real numbers. Given N stage cost indices L_j: $X\times U\rightarrow R_{\geq 0}$, $j=1,\ldots,N$. Define the cost function $J_j(\boldsymbol{x})$ as

$$J_j(\boldsymbol{x}(t_k))=\int_{t_k}^{t_k+T_\mathrm{p}}L_j(\boldsymbol{x}(s|t_k),\boldsymbol{u}(s|t_k))\mathrm{d}s \tag{3}$$

where number $0<T_\mathrm{p}<\infty$ is a prediction horizon, variables $\boldsymbol{x}(s|t_k)$ and $\boldsymbol{u}(s|t_k)$ are the values of states and controls at time s predicted at time t_k, respectively, and $\boldsymbol{x}(t_k)$ is the state at current time t_k, with $\boldsymbol{x}(t_k|t_k)=\boldsymbol{x}(t_k)$. In standard MPC, the goal is to asymptotically stabilize the origin of system (1), while minimizing a single performance function $J_j(\boldsymbol{x})$. The finite horizon optimal control problem of the standard MPC can be formulated as

$$\min_{\boldsymbol{u}(T_\mathrm{p},t_k)} J_j(\boldsymbol{x}(t_k))$$
$$\text{s.t.}\ \begin{cases}\dot{\boldsymbol{x}}(s|t_k)=\boldsymbol{f}(\boldsymbol{x}(s|t_k))+\boldsymbol{g}(\boldsymbol{x}(s|t_k))\boldsymbol{u}(s|t_k),\\ (\boldsymbol{x}(s|t_k),\boldsymbol{u}(s|t_k))\in Z,\\ \boldsymbol{x}(t_k|t_k)=\boldsymbol{x}(t_k),\quad \forall t_k\leq s\leq t_k+T_\mathrm{p}\end{cases} \tag{4}$$

where $\boldsymbol{u}(T_\mathrm{p},t_k)$ is the predictive control profile over the prediction horizon window $[t_k, t_k+T_\mathrm{p}]$ at sampling time t_k.

Remark 1. In this work, the states of system (1) are assumed to be sampled at each sampling time of the time sequence $\{t_k\}$ with $t_k=t_0+k\delta$ where the discrete-time index $k=0,1,\ldots$ and $\delta>0$ is the sampling period. Consequently, the control law obtained by minimizing the cost function (3) is applied to the continuous-time system (1) in a fashion of sampling-and-hold with the sampling period δ. For simplicity, let $t_0=0$.

Denote the optimal state and control trajectories obtained by solving optimization problem (4) as $\boldsymbol{x}_j^*(T_\mathrm{p},t_k)=\boldsymbol{x}_j^*(s|t_k)$ and $\boldsymbol{u}_j^*(T_\mathrm{p},t_k)=\boldsymbol{u}_j^*(s|t_k)$ for $t_k\leq s\leq t_k+T_\mathrm{p}$, respectively, where subscript j indicates the optimal state and control trajectories obtained by minimizing the jth cost function $J_j(\boldsymbol{x})$. In this MPC setup, the MPC law is defined in the usual receding horizon fashion: only the first part of the computed optimal trajectory $\boldsymbol{u}_j^*(s|t_k)$ up to the next sampling time $t_{k+1}=t_k+\delta$ is implemented to system (1), *i.e.*,

$$\boldsymbol{u}(s)=\boldsymbol{u}^*(s|t_k),\quad \forall t_k\leq s\leq t_{k+1}. \tag{5}$$

Then the optimization problem (4) is resolved at the next time t_{k+1}. Similar to standard MPC, the asymptotic stability of the closed-loop systems (1) and (5) cannot be guaranteed by the optimality of finite horizon cost functions (3). In addition, the recursive feasibility of Eq. (4) may not be ensured if the cost functions (3) are switched.

In this work, we use the dual-mode control approach and control Lyapunov function technique to achieve the asymptotic stability of the origin of the closed-loop system, while achieving the recursive feasibility of problem (4) for arbitrary switches between cost functions (3). In the following, some well-known notions and results are recalled in order to present our main results.

Definition 1. [23]

Consider a set $S\subseteq R^n$ and a constrained controller $\boldsymbol{u}$: $S\rightarrow U$. Set S is called a positively invariant set of the closed-loop system (1) with $\boldsymbol{u}(\boldsymbol{x})$ if its solution $\boldsymbol{x}(t;\boldsymbol{x}(0),\boldsymbol{u}(\boldsymbol{x}))\in S$ for any $t\geq 0$ and any $\boldsymbol{x}(0)\in S$.

Definition 2. [22]

Consider system (1) and a positive definite function $V(\boldsymbol{x})$. If $V(\boldsymbol{x})$ satisfies

$$L_{\boldsymbol{g}}V(\boldsymbol{x})=0\Rightarrow L_{\boldsymbol{f}}V(\boldsymbol{x})\leq 0,\quad \forall \boldsymbol{x}\neq 0 \tag{6}$$

where the Lie derivatives $L_{\boldsymbol{f}}V(\boldsymbol{x})=\frac{\partial V(\boldsymbol{x})}{\partial \boldsymbol{x}}\boldsymbol{f}(\boldsymbol{x}), L_{\boldsymbol{g}}V(\boldsymbol{x})=\frac{\partial V(\boldsymbol{x})}{\partial \boldsymbol{x}}[\boldsymbol{g}_1(\boldsymbol{x})\ \cdots \boldsymbol{g}_m(\boldsymbol{x})]$, and $\boldsymbol{g}_i(\boldsymbol{x})$, $i=1,\ldots,m$ is the ith column of matrix function $\boldsymbol{g}(\boldsymbol{x})$. Then $V(\boldsymbol{x})$ is said to be a control Lyapunov function (CLF) of the system. Moreover, if $V(\boldsymbol{x})\rightarrow\infty$ as $\|\boldsymbol{x}\|\rightarrow\infty$, $V(\boldsymbol{x})$ is a global CLF of the system.

Lemma 1. [24]

Let $V(\boldsymbol{x})$ be a CLF of system (1). For given numbers $D_1>0$ and $D_2>0$, there is a controller $\boldsymbol{u}(\boldsymbol{x}):=\boldsymbol{h}(\boldsymbol{x},\boldsymbol{\mu})$

$$\boldsymbol{h}(\boldsymbol{x},\boldsymbol{\mu})=-\kappa(\boldsymbol{x},\boldsymbol{\mu})\boldsymbol{\beta}(\boldsymbol{x})^\mathrm{T} \tag{7}$$

where free-parameters $\boldsymbol{\mu}=(\mu_1,\mu_2)\in(0,D_1]\times(0,D_2]$, $\alpha(\boldsymbol{x})=L_{\boldsymbol{f}}V(\boldsymbol{x})$, $\boldsymbol{\beta}(\boldsymbol{x})=L_{\boldsymbol{g}}V(\boldsymbol{x})$, and 'gain'

$$\kappa(\boldsymbol{x},\boldsymbol{\mu})=\begin{cases}\dfrac{\alpha(\boldsymbol{x})+\mu_1\sqrt{\alpha(\boldsymbol{x})^2+\mu_2\|\boldsymbol{\beta}(\boldsymbol{x})\|^4}}{\|\boldsymbol{\beta}(\boldsymbol{x})\|^2}, & \boldsymbol{\beta}(\boldsymbol{x})\neq 0\\ 0, & \boldsymbol{\beta}(\boldsymbol{x})=0\end{cases}$$