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A vibrational approach to determine the elastic modulus of individual thin films in multilayers

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ABSTRACT

A vibrational approach is presented to determine the elastic modulus of individual thin films deposited over a thicker substrate in multilayered systems. The approach requires measurement of the fundamental frequency of the multilayer and a laminated beam model for data reduction. A one-dimensional model based on classical laminated beam theory is introduced to provide a simple analytical approximation of the natural frequency of thin multilayered materials deposited over a significantly thicker substrate in cantilever beam configuration. The model has the advantage of providing an easy-to-use analytical expression for the natural frequency of a multilayered beam in terms of the elastic moduli of each layer, which can be inverted to calculate the elastic modulus of any individual layer if the elastic modulus of the remaining layers is known, and the natural frequency of the multilayered beam is measured. The limits of applicability of the proposed model are investigated by comparing its predictions of the fundamental frequency to those of an existent analytical model for bilayers and finite element analysis of materials comprising two and three dissimilar layers. The proposed model is applied to obtain the elastic modulus of Al and Au thin films in an Al/Au/Kapton multilayer.

1. Introduction

Laminated construction, where several dissimilar materials are stacked through the thickness of an integral structure, is a material architecture that has proven beneficial in many aspects [1–4]. The thickness of the layers can be in the order of millimeters or hundreds of micrometers forming a structure of macroscopic dimensions such as in the case of structural composite materials [5], or in the nanometric range such as in nano- and micro-electronic devices [1–4]. Designing layered structures at the nanoscale is an attractive strategy for developing multifunctional materials. Nano- and micro-metric thin multilayers find applications as wear and environment-protective coatings, magnetic data storage, electronic packaging, thermal barriers, biological, optical and electronic systems, among others [1–4,6–9]. The physical properties of such thin layers, however, may be different to those of their bulk counterparts, mainly because of their much higher ratio of surface area-to-volume and dramatically smaller length scale [1, 10–14]. Knowledge of the elastic behavior of a multilayer as a whole as well as of each individual layer in the multilayer is of particular importance to assess the mechanical behavior of the multilayered structure and to design for a reliable and safe operation. However, the elastic modulus of the individual layers in a multilayered structure may be different to that of an isolated layer of the same material and the same thickness (mainly because of interactions among the adjacent layers) and such an in-situ elastic modulus is quite challenging to measure. Among the methods used to determine elastic modulus in bilayers, the so called "vibration reed" method is one of the less intrusive and hence most convenient ones [15–17]. In this method, the elastic modulus of one layer in a bilayer is calculated from the shift in natural (or resonant) frequency of the beam without and with the second layer, which represents the input for an adequate model for data reduction [15–19]. Such a model provides an analytical expression for the fundamental frequency of the bilayer as a function of the fundamental frequency of the substrate and the elastic properties of both layers, so the problem formulation can be inverted to obtain the elastic modulus of the layer of interest. Although a few analytical solutions exist for the problem of determination of natural frequency of bilayers in thin film geometry [16, 20], the situation for more than two layers is fairly more complex and it often requires the use of more advanced laminated theories [21,22]. The investigation of the advantages and limitation of classical laminated theory for the study of the mechanics of laminated composites can be traced back to the early works of Pagano [23,24]. Pagano found agreement between classical laminated plate theory and elasticity solutions when the in-plane dimensions of the laminated plates are significantly larger than their thickness, and transverse (interlaminar) shear stresses can be neglected. This laminated theory has been extended over the years to elegant higher order theories and rigorous elasticity solutions

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describing the mechanics of layered composites, including vibrations [25–34]. However, these advanced laminated theories are often mathematically complex and their solutions frequently lack of a closed-form analytical expression, which is needed herein for the practical implementation of the vibratory model. To predict the fundamental frequency of multilayers, simple one-dimensional solutions, with a closed-form expression which is ready to use as a data reduction method for the frequency measurements seems to be inexistent. Given this motivation, this work presents a vibrational approach to obtain the elastic modulus of individual thin films forming part of a thicker multilayered structure. The main contribution of this work is to propose an integral approach for measuring such an elastic modulus, combining dedicated vibrational experiments and an ad-hoc, simple, and easy-to-use vibrational model. As the model for data reduction, the research proposes the use of a closed-from vibrational model based on one-dimensional classical laminated beam theory to predict the fundamental frequency of a multilayered structure as a function of the elastic properties of the constituent layers. The solution is first benchmarked against existent vibratory solutions for monolithic beams and bilayers, as well as finite element analysis, and then extended to the case of multilayers. Since the model assumes that the layered beam is symmetric (with respect to its thickness), limitations of the proposed laminated solution regarding the maximum allowed thickness of the films in the multilayer structure are examined, and estimations of the incurred error are provided considering the numerical finite element solution as reference. Finally, the model is used to obtain the elastic modulus of aluminum (Al) and gold (Au) thin films in a multilayer system composed of an Al/Au bilayer sequentially deposited over a polyimide (Kapton) substrate.

2. Vibrational modeling

The vibrating beam considered herein is a slender cantilever beam of length l, width b and total thickness $h = \sum_{k=1}^{n} h_k$ as shown in Fig. 1a. In its most general form, the beam is constituted of n dissimilar layers each one indicated by the ply index k ($k = 1\ldots n$), Fig. 1b. The thickness of each layer is h_k and the beam's mid-plane is located at $z = 0$. Closed-form solutions for the fundamental frequency of such a beam for simplified cases with one or two layers exist in the literature [16,20,34]. However, the solution for the natural frequency of a laminated beam of several (>2) layers is rather complex. Solutions to this vibratory problem exist, see e.g. [27,29], but they demand high order theories or elasticity solutions which inhibit its practical use as a straightforward data reduction method to this problem. Therefore, a simplified one-dimensional model based on classical laminated theory [5,23,24] is developed here for a symmetric laminate with an arbitrary number of layers. Known solutions for beams of one and two layers are presented in Sections 2.1 and 2.2, while the proposed model for multilayers is presented in Section 2.3.

2.1. Euler–Bernoulli model for monolithic beams

For the case of a single-layer ($n = 1$) isotropic beam with slender geometry, cross-sectional area A, mass density ρ, elastic modulus E, and moment of inertia I_z subjected to transverse (z) vibrations, the well-known Euler–Bernoulli governing differential equation for the transverse displacements w is [34],

$$c^2 \frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} = 0 \tag{1a}$$

with,

$$c = \sqrt{\frac{EI_z}{\rho A}}. \tag{1b}$$

For the case of a cantilever beam with rectangular cross-section the solution for the fundamental vibrational frequency (f_0) is [34],

$$f_0 = \frac{0.1615h}{l^2}\sqrt{\frac{E}{\rho}} \tag{1c}$$

where l is the beam's length and h its thickness.

2.2. Whiting's bilayer model

The case of an anisotropic bilayer ($n = 2$) can be addressed through laminated theory or elasticity formulations [25,27,29]. However, since a closed-form analytical solution for the bilayer problem is needed here, the simplified solution presented by Whiting et al. [16] for isotropic bilayers is of particular interest. This solution is especially suited for thin films and coatings, which assumes that the beams are slender, isotropic, and that the effects of shear deformation and inertia are neglected. Using an energy variational principle, Whiting et al. proposed a solution for the natural frequency of the bimaterial (f_{bim}) normalized by that of the substrate (f_s) as [16],

$$\left(\frac{f_{bim}}{f_s}\right)^2 = \left[\frac{(1+E_r h_r)\left(1+E_r h_r{}^3\right)+3E_r h_r (h_r+1)^2}{(1+\rho_r h_r)(1+E_r h_r)}\right] \tag{2}$$

where the elastic modulus, thickness, and density ratios are given by $E_r = E_f / E_s$, $h_r = h_f / h_s$ and $\rho_r = \rho_f / \rho_s$. The subscripts "f" and "s" correspond to "film" (f) and "substate" (s).

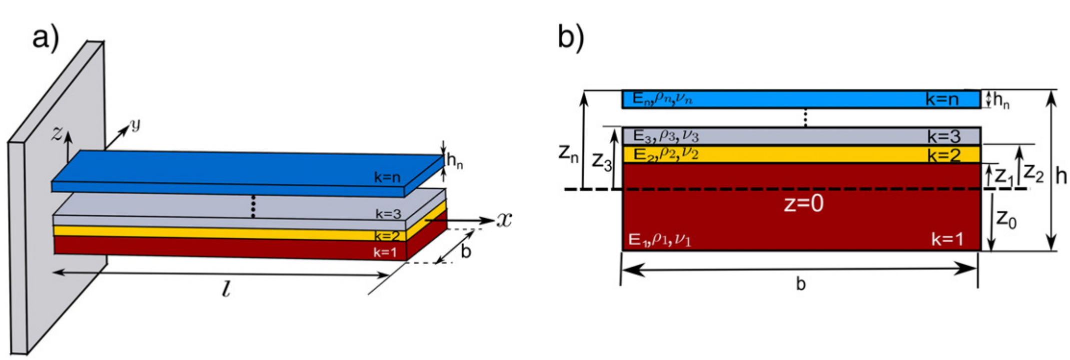

Fig. 1. Schematic representation of a multilayer beam. a) A multilayer beam in cantilever configuration, b) cross-section indicating the z-coordinates of each layer.