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Validation of analytical expressions for turbulent burning velocity in stagnating and freely propagating turbulent premixed flames

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ABSTRACT

A general expression is derived for the turbulent burning velocity, S_T, from the continuum form of the $\bar{c}$ transport equation and shown to be valid in all turbulent premixed combustion regimes. It involves the inverse length scale, $1/L_w$, for which new analytical relationships are proposed in the laminar flamelet and the distributed reaction regime. They are combined to give new predictive relationships for the S_T in the two limiting regimes and extended to be applicable in the intermediate regime as well. They involve flamelet thickness, mean curvature, molecular and turbulent diffusivities at the leading edge without any tuning constants. The proposed relationships are shown to be consistent with measurements in literature at varying pressures, laminar flame speeds, turbulent intensities and mixture compositions. Convincing agreement is achieved for S_T and $1/L_w$ for different turbulence and laminar flame properties of stagnating compressible flames and in parametric study with respect to turbulent intensity, laminar flamelet thickness and integral length scale for freely propagating incompressible flames. There is no gravity and the Lewis number is assumed unity for simplification.

1. Introduction

The turbulent burning velocity, S_T, is a parameter of crucial engineering importance to represent the mean reaction rate in turbulent premixed combustion. Here it is defined as the turbulent displacement speed as an analogue of the laminar displacement speed relative to convective flow at the leading edge. It may be different from the turbulent consumption speed defined as the burning rate integrated in the direction normal to the mean flame orientation in diverging or converging flow. The turbulent consumption speed has been considered physically a more meaningful quantity by some investigators, but requires more detailed information on flame surface density (FSD) and brush thickness which are not available in most measurements [1]. Although there have been quite a few data and correlations for the S_T in literature, they are known to involve excessive scatters and uncertainties in different flame configurations. It was suggested to group premixed flames into several categories and to make meaningful comparison in each category, e.g. V-flames, Bunsen, spherical and stagnating flames [2]. The global flow characteristics become independent of the *Re* in the limit of a large *Re* in cold flow turbulence according to Kolmogorov hypotheses, but there are no conclusive evidences to support such a premise in turbulent premixed combustion [3]. The laminar flame characteristics and the Lewis number remain as an important factor to determine the S_T both in small scale turbulence and large scale turbulence [4,5]. There were also some efforts to understand the effects on the S_T of various instability mechanisms including diffusive-thermal, hydrodynamic and buoyancy driven Rayleigh–Taylor instabilities separately from those of turbulence [6].

Recently an analytical expression was proposed for the S_T of a one dimensional turbulent premixed flame as [7,8]

$$S_T = D_{tu}/L_w + I_0 S^0_{Lu} \quad (1)$$

from the asymptotic behavior of the mean reaction progress variable at the leading edge. S^0_{Lu} is the unstretched laminar flame speed, D_{tu} is turbulent diffusivity and I_0 is the mean stretch factor. Although derivation was based on the Heaviside function for an infinitesimally thin flamelet, it was shown valid in a wide range of finite thickness flamelets and to be consistent with experimental correlations including those by Bradley et al. [9]. It was validated successfully for DNS of freely propagating flames [7] and flames stabilized in an impinging jet [8] with the factor, $1/L_w$, obtained from DNS data. The former simulation was performed in incompressible flow with no feedback from combustion to turbulence imposed on the mean flow. $1/L_w$ was termed the inverse characteristic scale of wrinkling at the leading edge, but with no further description or functional relationship provided. It was shown that $1/L_w$ was responsible for bending of the S_T curve at high turbulent intensities [7]. The factor,

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Nomenclature

A	pre-exponential factor for $\dot{\omega}$
c	reaction progress variable
Da	Damköhler number
D_m	molecular thermal diffusivity
D_{mu}	molecular diffusivity
D_{tu}	turbulent effective diffusivity
E_a	activation energy
FSD	flame surface density
H	heaviside function
I_0	mean stretch factor
Ka	Karlovitz number
ℓ_G	Gibson length scale
ℓ_t	turbulent integral length scale
Le	Lewis number
L_m	unstretched laminar flame thickness
L_m^*	flamelet thickness broadened by turbulence
L_w	characteristic scale of flame wrinkling at the leading edge
$\mathbf{n}$	unit normal vector to a flame surface
P	pressure
R_0	universal gas constant
Re	Reynolds number
Re_t	turbulent Reynolds number
S_d	displacement speed
S_{Lu}^0	unstretched laminar flame speed
S_T	turbulent burning velocity
t	non-dimensional time
T	non-dimensional temperature
T_a	activation temperature
T_b	burned gas temperature
T_u	unburned gas temperature
u'	turbulent intensity
U	mean flow speed
$\mathbf{v}$	velocity vector
Y_R	reactant mass fraction
Greek letters	
δ	dirac delta function
δ_L^0	effective laminar flame thickness in terms of the maximum temperature gradient
ϕ	arbitrary scalar or vector quantity
η	Kolmogorov length scale
ρ	non-dimensional density
ρ_b	burned gas density
ρ_u	unburned gas density
τ_c	characteristic chemical time scale
τ_t	turbulent integral time scale
τ_η	Kolmogorov time scale
$\dot{\omega}$	chemical reaction rate
Σ_f	generalized FSD
Σ_f'	instantaneous local FSD
Σ_g	geometric FSD
Σ_g'	instantaneous local geometric FSD
Averaging	
$\bar{\cdot}$	unconditional averaging
$\langle\cdot\rangle$	unconditional averaging
$\langle\cdot\rangle_f$	surface averaging with respect to generalized FSD
$\langle\cdot\rangle_g$	surface averaging with respect to geometric FSD

$1/L_w$, was affected by both turbulence and mean strain rate, while D_{tu} was affected by turbulence only in stagnating flames [8]. It was due to difference in $1/L_w$ that the S_T defined at the leading edge was different in a stagnating flame from that in a freely propagating flame under the same upstream turbulence and laminar flame conditions. In this paper we consider the continuum and the wave propagation forms of transport of the mean reaction progress variable to derive S_T and $1/L_w$ in the limiting conditions of distributed reaction and laminar flamelet regimes. They are shown to be applicable in a wider range of the intermediate regime in their limiting forms of the distributed reaction regime and in an extended form in terms of broadened flamelet thickness near the laminar flamelet regime. Validation is performed for the new expressions of S_T and $1/L_w$ against available data in literature and DNS results of an extensive set of stagnating and freely propagating flames in this study.

2. Two expressions for the S_T from continuum and wave propagation form of the $\bar{c}$ transport equation

The reaction progress variable, c, may be defined in terms of temperature or a deficient reacting species in premixed combustion. A continuum form of the transport equation for c is derived from the energy conservation equation as

$$\frac{\partial}{\partial t}(\rho c) + \nabla\cdot(\rho\mathbf{v}c) = \nabla\cdot(\rho D_m \nabla c) + \rho\dot{\omega} \tag{2}$$

where $c = (T - T_u)/(T_b - T_u)$ with a constant specific heat. The subscripts u and b represent the properties in unburned and fully burned gas respectively. D_m represents molecular thermal diffusivity. It may be transformed into a wave propagation form as

$$\frac{\partial c}{\partial t} + \mathbf{v}\cdot\nabla c = \frac{1}{\rho}\nabla\cdot(\rho D_m\nabla c) + \dot{\omega} = S_d\Sigma_f' \tag{3}$$

where the displacement speed, S_d, and the local flame surface density (FSD), Σ_f', are defined as

$$S_d \equiv \frac{1}{|\nabla c|}\left(\frac{1}{\rho}\nabla\cdot(\rho D_m\nabla c) + \dot{\omega}\right) \tag{4}$$

$$\Sigma_f' \equiv |\nabla c| \tag{5}$$

The reacting field may be approximated as unburned and fully burned gas separated by a sharp interface at a high Damköhler number (Da), so that Eq. (3) may be approximated in terms of the Heaviside function as [10]

$$\frac{\partial H}{\partial t} + \mathbf{v}\cdot\nabla H = S_d\Sigma_g' \tag{6}$$

The Heaviside function is defined as

$$\begin{aligned} H(c - c^*; \mathbf{x}, t) &= 0 \quad \textit{for} \quad 0 \leqslant c < c^* \\ H(c - c^*; \mathbf{x}, t) &= 1 \quad \textit{for} \quad c^* \leqslant c < 1 \end{aligned} \tag{7}$$

and the local geometric FSD is defined as

$$\Sigma_g' = |\nabla c|\delta(c - c^*) \tag{8}$$

where c^* is an arbitrary choice between zero and unity through a flamelet. S_d is the displacement speed relative to convective flow on the interface at $c = c^*$. Here we choose c^* close to zero to define the flame surface in the convection–diffusion layer of a flamelet. It allows us to employ the molecular and turbulent transport properties in unburned gas, not affected by reaction and density variation. We obtain the generalized FSD by averaging Eq. (5) as $\langle\Sigma_f'\rangle = \Sigma_f$ and the geometric FSD by averaging Eq. (8) as $\langle\Sigma_g'\rangle = \Sigma_g$. Hereafter the overbar or $\langle\cdot\rangle$ without any subscript represents