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Nonlinear thermoacoustics of ducted premixed flames: The influence of perturbation convection speed

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ABSTRACT

When a premixed flame is placed within a duct, acoustic waves induce velocity perturbations at the flame's base. These travel down the flame, distorting its surface and modulating its heat release. This can induce self-sustained thermoacoustic oscillations. Although the phase speed of these perturbations is often assumed to equal the mean flow speed, experiments conducted in other studies and Direct Numerical Simulation (DNS) conducted in this study show that it varies with the acoustic frequency. In this paper, we examine how these variations affect the nonlinear thermoacoustic behaviour. We model the heat release with a nonlinear kinematic *G*-equation, in which the velocity perturbation is modelled on DNS results. The acoustics are governed by linearised momentum and energy equations. We calculate the flame describing function (FDF) using harmonic forcing at several frequencies and amplitudes. Then we calculate thermoacoustic limit cycles and explain their existence and stability by examining the amplitude-dependence of the gain and phase of the FDF. We find that, when the phase speed equals the mean flow speed, the system has only one stable state. When the phase speed does not equal the mean flow speed, however, the system supports multiple limit cycles because the phase of the FDF changes significantly with oscillation amplitude. This shows that the phase speed of velocity perturbations has a strong influence on the nonlinear thermoacoustic behaviour of ducted premixed flames.

1. Introduction

Models of thermoacoustic systems contain a model for the heat release rate and a model for the acoustics. This paper concerns the model for the heat release rate of premixed flames. One approach is to model the flame surface as a zero-contour of a continuous function *G*. The flame propagates normal to itself, while also being advected by the surrounding flow. This is known as the *G*-equation model and is often used in models of thermoacoustic systems, either directly [1–3] or in the form a kinematic flame-tracking equation derived from the *G*-equation model [4–8]. In this model, the velocity field influences the heat release rate by distorting the flame's surface.

The velocity field can be decomposed into a steady base flow and acoustic, vortical and entropy perturbations [9]. Acoustic waves induce velocity perturbations at the base of the flame by exciting a convectively unstable shear layer. These velocity perturbations then travel along the flame, distorting its surface and therefore causing flame area fluctuations that result in unsteady heat release rate oscillations.

Several studies have investigated in detail the receptivity of shear layers to acoustic forcing and the enhanced generation of vorticity in separated shear layers by acoustic forcing, but most of these studies were for non-reacting flows. These have been reviewed by Wu et al. [10]. Similar studies for reacting shear layers are fewer, but are receiving more attention in recent years [11–13]. While these studies have generated a wealth of information about the response of shear layers to acoustic forcing, measurements or calculations of the phase speed of shear layer disturbances are limited, especially in the reacting case.

Experiments on laminar premixed flames by Boyer and Quinard [14] show that the hydrodynamic feedback from the flame changes the flow upstream, makes the acoustic field non-uniform and in particular, alters the convective speed of vortices that distort the flame surface. Experiments on forced conical premixed flames by Baillot et al. show that the disturbance propagation speed depends on the frequency but is independent of spatial location [15]. Velocity field measurements of an oscillating bunsen flame by Ferguson et al. show that streamwise disturbances are transported at a speed different from the average flow velocity [16]. Birbaud et al. investigated flows in acoustically excited free jets and premixed conical

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Nomenclature

ϕ	equivalence ratio
$\tilde{u}$	streamwise velocity
$\tilde{v}$	transverse velocity
ω	angular frequency
k	wavenumber
c	phase speed
K	ratio of steady streamwise base flow velocity to streamwise phase speed, $\tilde{u}_0/c_x$
f_{exc}	forcing frequency
i	the complex number $\sqrt{-1}$
ϵ	velocity perturbation amplitude normalised by its steady value
$\Delta\phi$	velocity perturbation phase
R	slot half-width
L_f	nominal flame height
β_f	flame aspect ratio = L_f/R
s_L	laminar flame speed
s_{Lu}	unstretched laminar flame speed
T_u	unburned gas temperature
p	pressure
ρ	density
γ	ratio of specific heats
Y	mass fraction
W	molecular weight
L_u	Markstein length
St	Strouhal number = $2\pi f_{exc}\beta_f R/\tilde{u}_0$
G	level set (function of space and time), $G = 0$ represents the flame surface
A	instantaneous flame surface area
s_L	laminar flame speed
h_R	heat of reaction (per unit mass)
$\dot{Q}$	heat release rate
$\tilde{c}_0$	speed of sound
M	mach number
$\delta()$	Dirac delta function
L_0	length of duct
x_f	flame position in the duct normalised by length of duct
ζ	damping parameter
c_1	damping factor corresponding to acoustic radiation
c_2	damping factor corresponding to boundary layer losses
ω_1	fundamental acoustic frequency of the duct
ν	viscosity
χ	thermal conductivity
E	acoustic energy density
T	acoustic time period of one cycle of oscillation at the fundamental frequency
φ	phase of any quantity in a Fourier expansion
f^*	non-dimensional fundamental frequency of the self-excited system, $f^* = \tilde{c}_0 L_f/2u_0 L_0$

Modifiers

$\hat{(\cdot)}$	Fourier component at the forcing St
$(\cdot)_x$	streamwise quantity
$(\cdot)_y$	transverse quantity
$(\cdot)'$	perturbation about the steady value
$(\cdot)_0$	steady base flow value
$\lvert\cdot\rvert$	magnitude of perturbations or gain of a function
$\angle\cdot$	phase of perturbations or a function
$\tilde{(\cdot)}$	dimensional quantity
$(\cdot)^*$	non-dimensional quantity
$\langle\cdot\rangle$	time average of the quantity over a forcing cycle

flames and found different oscillatory regimes that depend on the forcing frequency [17,18]. In the convective regime they found that the perturbation convection speed depends on the forcing frequency and the distance from the burner exit plane but is independent of the forcing amplitude [17]. This is the most detailed study of the upstream flow dynamics of an acoustically forced bunsen flame. Durox et al. in their study of the acoustic response of various configurations of laminar premixed flames point out that an accurate description of the oscillating flow field, including the fluid motion in the shear layer region, is crucial to capture the flame response [19,20]. Kornilov et al. show using particle image velocimetry measurements in a multi-slit burner that travelling waves are generated just upstream of the burner exit plane due to acoustic forcing [21,22]. Karimi et al. show clearly from measurements of the perturbed flame surface of a forced bunsen flame that the disturbance convection speed along the flame is not equal to the mean flow [23]. They use their measured perturbation convection speeds to make more accurate estimates of the phase of the flame describing function.

Experiments on a slot burner by Kartheekeyan and Chakravarthy and experiments on rod-stabilized flames and bluff-body stabilized flames by Shanbhogue et al. and Shin et al. show that the vortices that roll up in the acoustically excited shear layer convect along the flame at a speed not necessarily equal to the mean flow and usually slightly lower than the mean flow [24–26,6]. O'Connor et al. study the hydrodynamics of transversely excited swirling flow fields, both non-reacting and reacting, and show that the amplification of acoustic disturbances by convectively unstable shear layers dominate the wrinkling and dynamics of the flame [27–29]. Due to the multi-dimensional disturbances in their configuration, however, it was too complicated to extract a single phase speed of perturbations.

It is clear from the above studies that the disturbance characteristics in acoustically excited shear layers have a significant impact on the flame response and in particular, the convection speed of perturbations depends on several parameters and is not equal to the mean flow velocity.

These velocity perturbations that distort the flame surface are usually assumed to take the form of a travelling wave exp[i$(kx - \omega t)$], for which the streamwise phase speed, c_x, is equal to ω/k. The ratio of the mean velocity, u_0, to this phase speed, c_x, is denoted K. Most models assume that the phase speed is equal to the mean flow velocity, *i.e.* that $K = 1$ [30–32]. Preetham et al. investigate the influence of the parameter K on the gain and phase of the transfer function and they conclude that nonlinear effects are enhanced for $K > 1$ [1]. They, however, do not measure the dependence of K on frequency and amplitude of the acoustic perturbations in their analysis.

Our previous work on a simple thermoacoustic system containing a G-equation model of a premixed flame showed that, if $K = 1$, the system has at most one limit cycle and that this limit cycle must be stable [33]. However, recent experiments on similar systems show that there can be several limit cycles and that some are unstable [34–36]. Our analysis [33] captured this behaviour when K was greater than 1, but we did not justify this. In this paper we justify the values of K for the simple thermoacoustic system of a ducted premixed flame. The aims of this paper are (i) to extract K from Direct Numerical Simulation (DNS) of a premixed flame in order to provide an improved velocity perturbation model for inclusion in the G-equation; (ii) to use the G-equation model to