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Transient growth of flow disturbances in triggering a Rijke tube combustion instability

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ABSTRACT

Combustion instabilities in a Rijke tube could be triggered by the transient growth of flow disturbances, which is associated with its non-normality. In this work, a Rijke tube with three different temperature configurations resulting from a laminar premixed flame are considered to investigate its non-normality and the resulting transient growth of flow disturbances in triggering combustion instabilities. For this, a general thermoacoustic model of a Rijke tube is developed. Unsteady heat release from the flame is assumed to be caused by its surface variations, which results from the fluctuations of the oncoming flow velocity. Coupling the flame model with a Galerkin series expansion of the acoustic waves present enables the time evolution of flow disturbances to be calculated, thus providing a platform on which to gain insights on the Rijke tube stability behaviors. Both eigenmodes orthogonality analysis and transient growth analysis of flow disturbances are performed by linearizing the flame model and recasting it into the classical time-lag $\mathcal{N}-\tau$ formulation. It is shown from both analyses that Rijke tube is a non-normal thermoacoustic system and its non-normality depends strongly on the temperature configurations and the flame position. Furthermore, the most 'dangerous' position at which the flame is more susceptible to combustion instabilities are predicted by real-time calculating the maximum transient growth rate of acoustical energy.

1. Introduction

Low NOx emissions can be achieved by premixing the fuel and air and burning at lean equivalence ratios. In order to meet more and more stringent emission requirements, lean premixed prevaporized combustion technology is widely applied in both land-based gas turbines and aero-engines. However, this technology is often associated with combustion instabilities, which are characterized by large-amplitude self-excited oscillations. The pressure oscillations may become so intense that they cause structural damage and costly mission failure [1,2].

Self-excited combustion oscillation is currently a major challenge for land-based gas turbine and aero-engine manufacturers. It also occurs frequently in other types of combustion systems, including ramjets, boilers and furnaces [1]. On a laboratory-scale, combustion instabilities can occur in Rijke tube [3,4], which is a simple open-ended vertical tube with a confined flame in its lower half. Rijke tube is a nonlinear [3,5,6] and nonnormal [7–9] thermoacoustic system. The non-linearity and non-normality are due to the presence of the heat source and its complex interaction with flow disturbances. The non-normality indicates that small flow perturbations may exhibit large transient energy growth, which might be able to trigger combustion instabilities.

In order to analyze the role of non-normality in triggering combustion instabilities, thermoacoustic models of a Rijke tube with an electrical heater confined were developed [7,10]. For convenience, the mean temperature in the pre-heated zone was assumed to be the same as that of after-heated zone, although it is not experimentally justified [11]. The model [7] was then adopted [8] to predict the most 'dangerous' initial state that can trigger combustion instabilities via applying an adjoint looping algorithm and a conjugate gradient algorithm. The same model was used to examine the noise effect on triggering a Rijke-type combustion instability [12]. It was found that pink noise is more effective than white noise and blue noise.

To prevent triggering of combustion oscillations in a Rijke tube and to analyze the role of non-normality in active control of combustion instabilities [13], a transient growth controller was developed and applied to the model [7], which was modified by considering the mean temperature gradient effect on mean flow properties (such as mean density, flow and sound speed). However, the effect of mean temperature gradient on the modes frequencies was not considered, even it greatly influences the dynamics and stability behaviors of the Rijke tube. Neither was included in the

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governing equations for the flow fluctuations, i.e. Eqs. (2.8) and (2.9).[1] Lack of these investigations partially motivates the present study.

A premixed laminar flame is often used as a localized heat source in a Rijke tube. The kinematic response of such flame to flow disturbances has been investigated by several researchers [14–17]. Of particular relevant to this work is the theoretical modeling work [14]. The dynamic response of a premixed laminar flame, burning inwards from a ring on the duct was considered. The classical G-equation describing the flame displacement is simplified, by assuming the slope of the displacement of the flame surface $d\xi/dt \gg 1$. An analytical solution is then obtained by implementing Laplace transform method. However, for a premixed flame with $d\xi/dt \sim \mathcal{O}(1)$, as observed in our experiment, the theoretical model [14] is not applicable.

In this work, we propose a generalized thermo-acoustic model of a Rijke tube, providing a platform on which to gain insights on transient growth of flow disturbances in triggering combustion instabilities. This involves the development of a nonlinear laminar premixed flame model. The flame dynamic response is described by using the classical G-equation [18,14] and the flow disturbances are expanded using Galerkin series [5,7,19]. In Section 2, the model equations are developed. In Section 3, the influences of the mean temperature gradient on the modes frequencies are discussed. Comparison is then made between the predicted frequencies from the present model, those from previous works [20,21] and those from the experiment. In Section 4, eigenmodes orthogonality analysis is performed. A Rijke tube with three different temperature configurations are considered: (1) without a heat source and mean temperature gradient (2) with a heat source but no mean temperature jump across the heat source, (3) with a heat source and a mean temperature gradient. Finally, in Section 5, transient growth analysis of the flow disturbances in the Rijke tube is conducted and compared with the results from eigenmodes orthogonality analysis. Furthermore, the most 'dangerous' positions at which the flame is more susceptible to combustion instabilities are predicted by real-time calculating the maximum transient growth rate of acoustical energy.

2. Description of the numerical model

The numerical model considers a conventional Rijke tube, which is a simple and widely studied example of self-excited combustion system. The Rijke tube is a straight open-ended tube, with a premixed flame inside. The numerical model captures two physical processes; the generation and propagation of acoustic waves within the Rijke tube and the response of the flame to acoustic waves.

2.1. Model of acoustic wave generation and propagation

The pressure of the gas mixtures used in the Rijke tube under consideration is always low enough to justify the assumption of the perfect gas laws. Thus if p denotes the instantaneous pressure, T the temperature and ρ the density,

$$p = \rho RT = \frac{\rho c^2}{\gamma} \tag{1}$$

where c is the speed of sound, $\gamma = 1.4$ is the ratio of specific heats, R is the gas constant per kilogram of the gas mixture.

If we denote the internal energy of the gas mixture by E, then

$$E = \frac{c_v}{R}\frac{p}{\rho} = \frac{1}{\gamma - 1}\frac{p}{\rho} \tag{2}$$

where c_v is the constant-volume specific heat. The first law of thermodynamics considering energy conservation can be described by

$$\frac{dE}{dt} + p\frac{d(1/\rho)}{dt} = \frac{\partial Q(T)}{\partial t} \tag{3}$$

The convection flow in Rijke tube has vanishingly small Mach number $\overline{M}_a$ [22,23], so that we can expand the flow properties in a region of the gas mixture where the mean pressure, temperature and density are $\bar{p}, \overline{T}$ and $\bar{\rho}$ respectively as

$$\frac{\rho}{\bar{\rho}} = 1 + \overline{M}_a\sigma_1 + \mathcal{O}(\overline{M}_a^2\sigma_2) + \cdots \tag{4}$$

$$\frac{T}{\overline{T}} = 1 + \overline{M}_a\mu_1 + \mathcal{O}(\overline{M}_a^2\mu_2) + \cdots \tag{5}$$

Substituting the linearized form of Eqs. (4) and (5) by neglecting the second and higher order terms into Eq. (3) leads to

$$(1 + \overline{M}_a\mu_1)\frac{d}{dt}\ln\left\{(1 + \overline{M}_a\mu_1)(1 + \overline{M}_a\sigma_1)^{1-\gamma}\right\} = \frac{\gamma - 1}{R\overline{T}}\frac{\partial Q(T)}{\partial t} \tag{6}$$

Application of Taylor series expansion on Eq. (6)[2] and neglecting second order terms gives rise to the linearized form of Eq. (6) as follow

$$\frac{\gamma - 1}{R\overline{T}}\frac{\partial Q(T)}{\partial t} = \frac{\partial}{\partial t}\left(\overline{M}_a\mu_1 + (1 - \gamma)\overline{M}_a\sigma_1\right) \tag{7}$$

The motion of the perfect gas can be described by the following equation

$$\frac{d\mathbf{V}}{dt} + \frac{1}{\rho}\nabla p = 0 \tag{8}$$

where the vector $\mathbf{V}$ denotes the gas velocity. Since the gas flow under consideration is irrotational, i.e. curl $\mathbf{V} = 0$, a velocity potential ϕ exists [24] as $\mathbf{V} = -\nabla\phi$. Substituting $\mathbf{V}$ with the velocity potential ϕ, replacing $d\mathbf{V}/dt$ with $\partial\mathbf{V}/\partial t + (\mathbf{V}\cdot\nabla)\mathbf{V}$ and using Eq. (1) yields

$$\nabla\frac{\partial\phi}{\partial t} - \nabla\left(\frac{1}{2}\mathbf{V}^2\right) = R\nabla T + \frac{RT}{\rho}\nabla\rho \tag{9}$$

The equation of mass conservation can be shown

$$\nabla\cdot(\rho\mathbf{V}) + \frac{\partial\rho}{\partial t} = 0 \tag{10}$$

Substituting Eqs. (4), (5) into Eq. (10) and replacing $\mathbf{V} = -\nabla\phi$ yields

$$\frac{d}{dt}\{\ln(1 + \overline{M}_a\sigma_1)\} = \nabla^2\phi \tag{11}$$

Use of Taylor series expansion and neglecting second and high order terms leads to

$$\frac{\partial(\overline{M}_a\sigma_1)}{\partial t} = \nabla^2\phi \tag{12}$$

Similarly, we can linearize Eq. (9) by neglecting second order terms of $\overline{M}_a\mu_1$ and $\overline{M}_a\sigma_1$ and such products as $\overline{M}_a\mu_1\nabla\overline{M}_a\sigma_1$, and integrating it, then we obtain[3]

[1] When there is no mean temperature gradient, i.e. $T_2 = T_1$, the pressure nodes at both open ends of the Rijke tube yields $\omega_j/\bar{c} = j\pi/L$, where $\bar{c}$ is speed of sound, L is the length of the tube and j is an integer. Thus $p'(x,t)$ can be formulated in terms of $\sin(j\pi\frac{x}{L})$. Normalizing x with L gives Eq. (2.8) [13]. Similar analysis yields Eq. (2.9). However, when $T_2 \neq T_1, \bar{c}_1 \neq \bar{c}_2$ and the mode frequency $\tilde{\omega}_j$ is different from the one ω_j determined by assuming $T_2 = T_1$. Thus the governing equations for the flow fluctuations need to include $\tilde{\omega}_j$ in the presence of mean temperature gradient.

[2] When $Q(T) = 0$ (i.e. no heat source), Eq. (6) can be simplified to $(1 + \overline{M}_a\mu_1) = (1 + \overline{M}_a\sigma_1)^{\gamma-1}$, which describes the general adiabatic law in terms of $\overline{M}_a\sigma_1$ and $\overline{M}_a\mu_1$.

[3] Note that any additional term which depends on the time and not on the space coordinates is neglected from ϕ.