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Generation of a spin-polarized electron beam by multipole magnetic fields

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ABSTRACT

The propagation of an electron beam in the presence of transverse magnetic fields possessing integer topological charges is presented. The spin–magnetic interaction introduces a nonuniform spin precession of the electrons that gains a space-variant geometrical phase in the transverse plane proportional to the field's topological charge, whose handedness depends on the input electron's spin state. A combination of our proposed device with an electron orbital angular momentum sorter can be utilized as a spin-filter of electron beams in a mid-energy range. We examine these two different configurations of a partial spin-filter generator numerically. The results of this analysis could prove useful in the design of an improved electron microscope.

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1. Introduction

A few years ago, the existence of electron beams carrying orbital angular momentum (OAM) was predicted theoretically [1]. A couple of years later, two different techniques, based on the holography and random phase changes in a graphite sheet, were used to generate electron beams carrying OAM, i.e., electron vortex beams, experimentally [2–4]. Such an intriguing topic is of particular interest to materials scientists, since it can be used to probe the magnetic and spin-dependent properties of materials with atomic resolution [5,6]. The OAM as a "*rotational-like*" degree of freedom of an electron beam induces a *magnetic moment*, in addition to the spin magnetic moment, of up to few hundred Bohr magnetons per electron, which gives a possibility to interact with an external magnetic field [3,5]. The interaction of OAM magnetic moments with a uniform longitudinal magnetic field or a flux has been recently examined theoretically and experimentally [7–9]. This interaction enhances or diminishes the kinetic OAM of the beam, and can be used to measure or sort the OAM of electron beams spatially. However, besides its interesting and fascinated applications, this novel degree of freedom of free electrons can be utilized to investigate some fundamental quantum concepts such as the *Bohr–Pauli impossibility of generating a spin-polarized free electron beam* [10,11].

The spin–orbit coupling in a non-uniform balanced electric-magnetic field, named a "*q*-filter", was proposed by some of the authors as a novel tool to generate an electron vortex beam from a pure spin-polarized electron beam. In that configuration, the spin of an electron follows the *Larmor precession* up and acquires a geometrical phase, which depends on both the spin–magnetic field direction and the time of interaction. A non-uniform magnetic field introduces a non-uniform phase profile whose topological structure is that of the magnetic field. Several different topological charge configurations, proposed in the previous paper, have been achieved in practice [11]. A locally orthogonal electric field was proposed to compensate for the "net" magnetic force. Furthermore, the reverse process was suggested to filter the spin component of an electron beam spatially, where two different longitudinal electron's spin components suffer opposite precession directions, thus possess opposite OAM values.

In this work, we suggest a scheme based on *non-uniform magnetic fields*, rather than a balanced space-variant *Wien q*-filter, to manipulate electron OAM. Analogous to the *q*-filter, the proposed scheme imprints a topological charge – identical to the topological charge of magnetic field – onto the incoming electron beam with a handedness depending on the longitudinal component of electron spin; positive for spin up $|{\Uparrow}\rangle$ and negative for spin down $|{\Downarrow}\rangle$ with the advantage that no compensating electric field is needed. Unlike in the *q*-filter, however, the beam structure is now

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strongly affected by the magnetic field. A TEM_{00} Gaussian beam splits out into *multi Gaussian-like* beams after passing through the nonuniform magnetic field of the device, each beam oscillates in the opposite direction of the gradient of A_z, i.e., along $-\nabla A_z$. In particular, the incident Gaussian electron beam splits into two and three semi-Gaussian beams in quadrupole and hexapole magnetic fields, respectively. It is worth noting, however, that a multi-Gaussian-like beam does recover its original Gaussian shape at certain free-space propagation distance, provided its phase distribution does not acquire sudden changes in the transverse plane. In this work, we introduce and numerically simulate two realistic configurations of a spin-filter for electron beams, based on the new proposed device. Our numerical simulations confirm that a portion of the electrons, typically small, remains polarized after passing through the device and can be easily separated form the rest of the beam by suitable apertures.

2. Propagation of electron beams in an orthogonal uniform magnetic field

Let us assume that the electron beam moves along the z-direction perpendicular to a uniform magnetic field $\boldsymbol{B}=B_0(\cos\,\theta, \sin\,\theta, 0)$, which lies in the (x,y) transverse plane at angle θ with respect to the x-axis. As associated vector potential we may take $\boldsymbol{A}=B_0(0,0,y\,\cos\,\theta-x\,\sin\,\theta)$. We assume a nonrelativistic electron beam, so that we can use Pauli's equation

$$i\hbar\,\partial_t\tilde{\psi}=\left\{\frac{1}{2m}(-i\hbar\nabla-e\boldsymbol{A})^2-\boldsymbol{B}\cdot\hat{\boldsymbol{\mu}}\right\}\tilde{\psi}, \tag{1}$$

where $\tilde{\psi}$ is a two-component spinor and $\hat{\boldsymbol{\mu}}=\frac{1}{2}g\mu_B\hat{\boldsymbol{\sigma}}$ is the electron magnetic moment, $\mu_B=\hbar e/2m$ is Bohr's magneton, g is the electron g-factor, and $\hat{\boldsymbol{\sigma}}=(\hat{\sigma}_x,\hat{\sigma}_y,\hat{\sigma}_z)$ is Pauli's vector, respectively. We assume a paraxial beam with average linear momentum p_c and average energy $E_c=p_c^2/2m$, so that $\tilde{\psi}(x,y,z,t)=\exp[i\hbar^{-1}(p_c z-E_c t)]\tilde{u}(x,y,z)$, with $\tilde{u}(x,y,z)$ a slow-envelope spinor field [1]. Inserting this *ansatz* into Eq. (1) and neglecting the second derivatives of $\tilde{u}$ with respect to z, we obtain the paraxial Pauli equation

$$\left\{2ik_c\partial_z+\nabla_\perp^2+2k_c\frac{e}{\hbar}A-\frac{e^2}{\hbar^2}A^2+\frac{2m}{\hbar^2}\boldsymbol{B}\cdot\hat{\boldsymbol{\mu}}\right\}\tilde{u}(x,y,z)=0, \tag{2}$$

where $\perp$ stands for the transverse coordinate, and $k_c=p_c/\hbar$ is the central de Broglie wave-vector.

Eq. (2) is solved with initial Cauchy data at $z=0$, $\tilde{u}(r,\phi,0)=\tilde{a}\,\exp(-r^2/w_0^2)$ corresponding to a Gaussian beam of width w_0 in the cylindrical coordinates of (r,ϕ,z). The constant spinor $\tilde{a}=(a_1,a_2)$ describes the polarization state $|\psi\rangle=a_1|\Uparrow\rangle+a_2|\Downarrow\rangle$ of the input beam in the $|\Uparrow\rangle$, $|\Downarrow\rangle$ basis where the spin is aligned parallel or antiparallel to the beam propagation direction, respectively. We assume the normalization $|a_1|^2+|a_2|^2=1$. A straightforward calculation shows that the required solution of the paraxial Pauli equation is given by

$$\tilde{u}(r,\phi,z)=G(r,\phi,z)\hat{M}(z)\tilde{a}, \tag{3}$$

where $\hat{M}(z)$ is a matrix given by

$$\hat{M}(z)=\begin{pmatrix}\cos\frac{2\pi z}{\Lambda_1} & ie^{-i\theta}\sin\frac{2\pi z}{\Lambda_1}\\ ie^{i\theta}\sin\frac{2\pi z}{\Lambda_1} & \cos\frac{2\pi z}{\Lambda_1}\end{pmatrix}, \tag{4}$$

with $\Lambda_1=4\pi\hbar^2k_c/mg\mu_B B_0$. The matrix $\hat{M}(z)$ accounts for the action of the magnetic field on the particle spin. The action on the electron motion is described in Eq. (3) by the Gaussian-coherent factor $G(r,\phi,z)$ given by

$$G(r,\phi,z)=\sqrt{\frac{-k_c z_R}{\pi q_\parallel(z)q_\perp(z)}}\,e^{ik_c(f_g(r,\phi)/2q_\parallel(z)+f_c(r,\phi,z)/2q_\perp(z)+z/2)} \tag{5}$$

with

$$f_g(r,\phi)=r^2\,\cos^2(\theta-\phi),$$

$$\begin{aligned}f_c(r,\phi,z)=&\,i\left(\frac{\pi}{\Lambda}\right)\left(\frac{\Lambda}{\pi}+r\,\sin\,(\theta-\phi)\right)\\&\times\left(2i\left(\frac{\Lambda}{\pi}\right)^2+\left(\frac{\pi}{\Lambda}\right)z_R\left(q_\perp(z)+iz_R\,\cos\left(\frac{\pi z}{\Lambda}\right)\right)\right.\\&\left.\times\left(\frac{\Lambda}{\pi}+r\,\sin(\theta-\phi)\right)\right)+\cos\left(\frac{\pi z}{\Lambda}\right)\\&\times\left(2\left(\frac{\Lambda}{\pi}\right)^2+r\,\sin(\theta-\phi)\left(2\left(\frac{\Lambda}{\pi}\right)+r\,\sin(\theta-\phi)\right)\right).\end{aligned} \tag{6}$$

The complex curvature radii of the Gaussian-coherent factor $G(r,\phi,z)$ are given by

$$q_\parallel(z)=z-iz_R \tag{7}$$

$$q_\perp(z)=\left(\frac{\Lambda}{\pi}\right)\,\sin\left(\frac{\pi z}{\Lambda}\right)-iz_R\,\cos\left(\frac{\pi z}{\Lambda}\right), \tag{8}$$

where $z_R=\frac{1}{2}k_c w_0^2$, $\Lambda=\pi\hbar k_c/eB_0$. From Eq. (4) we see that the beam spin state oscillates during propagation with spatial period Λ_1.

Moreover, unlike the case with the Wien-filter, in which the mean direction of the beam is not affected, now the beam oscillates perpendicularly to the magnetic field $\boldsymbol{B}$ towards $-\nabla A_z$ with spatial period $2\Lambda=(g/4)\Lambda_1\simeq\Lambda_1/2\propto B_0^{-1}$. Fig. 1(a) shows the beam intensity profiles at two different z-planes; at the entrance plane $z=0$ (central spot) and at $z=\alpha\Lambda$ with $\alpha=0.44\%$ (upper spot). The electron trajectory in the yz-plane, orthogonal to the magnetic field, is shown in Fig. 1(b): the electron beam follows a sinusoidal oscillation with spatial period 2Λ and amplitude $2\Lambda/\pi$.

3. Propagation of electron beams in an orthogonal nonuniform magnetic field possessing a specific topological charge

Eq. (3) represents an exact solution of the beam paraxial equation, with explicit boundary conditions, for a uniform constant magnetic field at angle θ with respect to the x-axis. If the angle $\theta=\theta(x,y)$ changes slowly in the transverse plane, we may assume that the solution (3) is still approximately valid. This Geometric Optics Approximation (GOA) is quite accurate in the present case, since the electron beam wavelength in a typical Transmission Electron Microscope (TEM) is in the range of tens of picometers, while θ changes over length of several microns. Within this slowly varying approximation, the effect of a nonuniform magnetic field is obtained simply by replacing θ with $\theta(x,y)$ in Eqs. (4)–(6). We assume a singular space distribution of the magnetic field where $\theta(x,y)=\theta(r,\phi)$ is given by

$$\theta(\phi)=q\phi+\beta, \tag{9}$$

where $\phi=\arctan(y/x)$ is the azimuthal angle in the beam transverse plane and β is a constant angle, which defines the inclination on the x-axis. Finally q is an integer which fixes the topological charge of the singular magnetic field distribution. Such magnetic structures can be generated in practice by multipolar lenses (for negative charges q) or by a set of appropriate longitudinal currents at origin (for positive

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