Ultramicroscopy 119 (2012) 40–44

Contents lists available at SciVerse ScienceDirect

Ultramicroscopy

journal homepage: www.elsevier.com/locate/ultramic

Aberrations in asymmetrical electron lenses ☆

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ARTICLE INFO

Available online 25 November 2011

Keywords:
Electron optics
Geometric optics
Electrostatic lens
Einzel lens
Asymmetric lens
Spherical aberration
Chromatic aberration

ABSTRACT

Starting from well established knowledge in light-optics we explore the question if electron-optical aberration can be improved in asymmetrical electron lenses. We show that spherical as well as chromatic aberration coefficients are reduced in asymmetric electrostatic einzel lenses when the center electrode is moved away from the center position towards the entrance electrode. Relative improvements up to 40% for both the chromatic and the spherical aberration coefficients can be obtained. We use analytical and numerical calculations to confirm this result for exemplary cases of a lens with fixed length and working distance. The agreement of the two calculation methods is very good. We then derive an estimate for the electron-optical aberration coefficients from light-optics. The derived expressions for chromatic and spherical aberrations are somewhat simpler than the ones derived from electron-optics as they involve integrals only over the electrostatic potential, not over the electron paths. The estimated formulas still agree well with the electron optical calculations. Overall, we are tempted to suggest that the enormous knowledge base of light optics can provide considerable guidance for electron-optical applications.

1. Introduction

Over the last centuries many qualitative and quantitative methods have been established in the field of light optics with the goal to optimize the fabrication of lenses with small aberrations. One well-known method to minimize spherical aberrations in a two-surface light-optical lens is the so-called bending of the lens. Essentially this bending consists of realizing an asymmetric design for the two refractive surfaces of the lens. Minimal spherical aberration is obtained when the shape of the lens satisfies the simple mathematical relation

$$-2p\left(\frac{n^2-1}{n+2}\right)=q,$$

with $p=(i+o)/(i-o)$,

and $q=(R_2-R_1)/(R_2+R_1)$, (1)

where R_1 and R_2 are the curvature radii of the lens entrance and exit surfaces, n is the refractive index, and o and i are the object and image distances, respectively. q is known as the Coddington shape factor, and p is the Coddington position factor. A derivation of Eq. (1) is found in standard optics books, such as [1,2].

Without going into the details or the proof of this interesting rule for light optics, we simply state that these formulas indicate that a magnifying lens should have smaller curvature radius on the object side. For example for glass with $n=1.5$ and for magnification $M=10$, we find that $R_2/R_1=-2$. From Fig. 1 we note that a magnifying lens should be bent towards the image side. Conversely, a de-magnifying lens should be bent to the object side and a transfer lens with $M=1$ should be symmetric. In these cases the bending of the lens ensures that after the first refraction the ray remains close to the optical axis, in effect it is kept approximately parallel to the axis between the two surfaces.

Trajectory electron optics can be formulated in very close analogy to light ray optics using Fermat's principle and the least action principle, respectively [3–5]. This intimate correspondence between light and electron optics raises the question if some of the wisdom generated and collected over the centuries in light optical design can successfully be carried over to electron optics. Thus one may anticipate that asymmetry could afford a possibility for a fine tuning of electron lenses. If the general result from light optics translates, one would expect the best electron lens to have a stronger field close to the object-side. It is apparent, however, that in a typical electron lens the electron trajectory is usually more complicated than the ray trace in a light-optical lens, as the electron optical medium is typically non-uniform and often anisotropic. In a multi-electrode, electrostatic lens an electron trajectory also typically comprises converging and diverging sections within a single lens.

In this paper we show that indeed an asymmetrical lay-out of electron optical einzel lenses offers the possibility to significantly

☆ In memoriam Professor Gertrude Rempfer on the occasion of her 100th birthday. The authors have thoroughly enjoyed her enlightening presence at PSU, her outstanding spirit as a scientist, and her generosity as a teacher.

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doi:10.1016/j.ultramic.2011.11.014

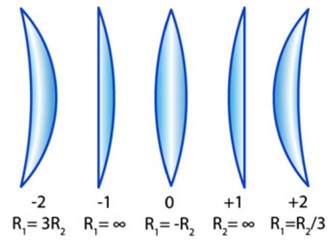

Fig. 1. "Bending" the lens in light optics. The object is on the left, the image to the right of the lens. Numbers in the first line represent the Coddington shape factor; the second line shows the optimal radii for the two refractive surfaces.

lower the spherical aberration. Such a conclusion has indeed been suggested earlier by [6] based on experimental lens characterization work. Surprisingly, the same asymmetric lens lay-out that minimizes spherical aberration also optimizes the chromatic aberration. This finding may appear more fortuitous, as the physical origin for chromatic aberration in light and electron optics seems not quite comparable. Nonetheless, the possibility of combining lower spherical and chromatic aberrations emphasizes the possible advantages of asymmetric lens designs in electron optics. The basic conclusion from this study is that for an einzel lens with a given length, the lay-out should be such that the center electrode is moved away from the symmetrical position towards the entrance electrode. Within the limits of our work, this conclusion seems to hold for all magnifications.

2. Lens description

For our calculations we utilize an analytical approach based on standard integral expressions for the aberration coefficients [7] and a numerical approach using a finite difference method. We carry out the calculations for an electrostatic, decelerating, three-electrode einzel lens as typically used in electron microscopes. We define the asymmetry of the lens by the ratio s_o/ℓ, where s_o is the distance between the entrance and the center electrode and ℓ is the length of the lens.

We present two cases of asymmetric lenses: in the first case the lens has a fixed length and is located at a fixed working distance from the object point. Here we define the working distance between the object point and the object-side electrode. Asymmetry in this case is implemented by moving the center electrode between the two grounded electrodes. Necessarily when the magnification or the center electrode position is changed, the image position is also changed. In the second case, the lens is also of fixed length, but now the center electrode position and the object position are kept constant. Thus in this case asymmetry is implemented by shifting entrance and exit electrodes together in relation to the center electrode and the object point, roughly maintaining the object distance of the lens. Again the image position changes with magnification and asymmetry. As magnification has a strong effect on the aberration coefficients, cases for different M are separately compared.

In our example the electrodes have apertures of equal diameter $d=0.1\ell$, where ℓ is the length of the lens, and the electrodes are of minimal thickness $t=0.01\ell$. This relates to a 1-cm long lens with 1-mm diameter apertures and 0.1-mm thick electrodes. The minimum spacing between electrodes at 20 kV for this "realistic" lens due to high voltage breakdown is 1–2 mm, so we consider electrode spacings between 0.15ℓ and 0.85ℓ. For the fixed working distance case the spacing between the point source object and the first electrode is kept constant at 0.5ℓ, which is similar to the minimum working distance of 4–10 mm typical of an electrostatic objective lens. For the second case, the distance between the center electrode and object point is maintained at ℓ.

3. Calculation of aberrations

We compute the aberrations analytically from

$$C_s=\frac{1}{16r_i'^4\sqrt{\phi_i}}\int_o^i r^2\sqrt{\phi}\left\{\left[\frac{5}{4}\left(\frac{\phi''}{\phi}\right)^2+\frac{5}{24}\left(\frac{\phi'}{\phi}\right)^2\right]r^2 +\frac{14}{3}\left(\frac{\phi'}{\phi}\right)^3 r'r-\frac{3}{2}\left(\frac{\phi'}{\phi}\right)^2(r')^2\right\}dz, \tag{2}$$

$$C_c=\frac{\sqrt{\phi_i}}{r_i'^2}\int_o^i\frac{r}{\sqrt{\phi}}\left[\frac{1}{2}\frac{\phi'}{\phi}r'+\frac{1}{4}\frac{\phi''}{\phi}r\right]dz, \tag{3}$$

given by Munro in [7], where z is the optical axis, r is a paraxial ray that crosses the z-axis at the object point o and again at the image point i, r_i' is the slope of that ray at the image point, ϕ is the electrostatic potential distribution on the z-axis, and ϕ_i is the potential at the image point. An analytic function,

$$\phi(z)=V_C+\frac{V_L}{\pi}\left[-\left(\frac{z+s_o}{s_o}\right)\tan^{-1}\left(\frac{z+s_o}{d/2}\right) +\left(\frac{z}{s_o}+\frac{z}{\ell-s_o}\right)\tan^{-1}\left(\frac{z}{d/2}\right) -\left(\frac{z-(\ell-s_o)}{\ell-s_o}\right)\tan^{-1}\left(\frac{z-(\ell-s_o)}{d/2}\right)\right], \tag{4}$$

derived by [8,9], is used to calculate the potential distribution and its derivatives as well as a numerical solution for the paraxial ray trajectory. In Eq. (4), V_L and V_C are the center electrode and accelerating potentials.

In the numerical approach the aberration coefficients are computed in SIMION 8, which uses a self-adjusting relaxation method for the potential and a cylindrically symmetric mesh with 160 points per mm. The mesh extends radially more than 2ℓ off axis and more than ℓ in either direction along the optical axis. The gradient of the potential is recursively refined to an accuracy of 5×10^{-5}. Twenty electron trajectories with angles less than 10 mrad are traced through the potential distribution with an adaptive algorithm that recalculates the rays at least once per mesh point. The center electrode potential is varied by $\Delta V_L/V_C=5\ \text{V}/20\ \text{kV}$ increments, where V_L is the center electrode potential and V_C is the electron accelerating potential. The paraxial magnification and spherical aberration are calculated at each step from the image point trajectories. Chromatic aberration is calculated from the change in potential along with the change in paraxial image distance via the relationship $C_c=-\Delta z/(\Delta V_L/V_L)$. Finally, the values of s_o are chosen randomly.

Angular magnification $M=\alpha/\alpha'$ is used for the magnification standard, where α is the object side trajectory angle and α' is the image side angle. Three different magnifications are examined, M=1, 2, and 8. These magnifications may be taken to represent lenses with different functions, such as transfer lenses with magnification 1 and projection and objective lenses with larger magnifications. The calculations are symmetric under interchange of object and image point, so the results can be adapted to a demagnifying-(focusing-) type objective lens. The center electrode potential is adjusted to maintain this magnification for the various lens asymmetries. All aberration coefficients were normalized with respect to the aberration in the symmetric case, $s_o/\ell=0.5$.