Ultramicroscopy 120 (2012) 41–47

Contents lists available at SciVerse ScienceDirect

Ultramicroscopy

journal homepage: www.elsevier.com/locate/ultramic

Modal interactions of flexural and torsional vibrations in a microcantilever

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ARTICLE INFO

Article history:
Received 14 March 2012
Received in revised form
4 June 2012
Accepted 12 June 2012
Available online 23 June 2012

Keywords:
Nonlinear dynamics
Cantilever
Modal interactions
AFM

ABSTRACT

The nonlinear interactions between flexural and torsional modes of a microcantilever are experimentally studied. The coupling is demonstrated by measuring the frequency response of one mode, which is sensitive to the motion of another resonance mode. The flexural–flexural, torsional–torsional and flexural–torsional modes are coupled due to nonlinearities, which affect the dynamics at high vibration amplitudes and cause the resonance frequency of one mode to depend on the amplitude of the other modes. We also investigate the nonlinear dynamics of torsional modes, which cause a frequency stiffening of the response. By simultaneously driving another torsional mode in the nonlinear regime, the nonlinear response is tuned from stiffening to weakening. By balancing the positive and negative cubic nonlinearities a linear response is obtained for the strongly driven system. The nonlinear modal interactions play an important role in the dynamics of multi-mode scanning probe microscopes.

1. Introduction

The Atomic Force Microscope (AFM) [1] is a crucial instrument in studying nanoscale objects. Various operation schemes are employed, which include the use of different cantilever geometries, higher modes or the torsional mode for imaging [2–5]. The nonlinear tip-sample interactions determine the dynamics in tapping-mode AFM and have been studied in detail [6,7]. Besides this extrinsic nonlinearity, the intrinsic mechanical nonlinearities determine the dynamics of ultra-flexible microcantilevers at high amplitudes, as shown in a recent study [8]. These nonlinearities result in an amplitude-dependent resonance frequency and couple the vibration modes. In clamped–clamped beams, the nonlinear coupling is provided by the displacement-induced tension [9,10]. For cantilever beams it was shown that the coupling between the modes can be used to modify the resonance linewidth [11]. In a multi-mode AFM [12,13], these modal interactions are of importance, since the resonance frequency of one mode depends on the amplitude of the other modes.

In this work, we experimentally demonstrate the intrinsic mechanical coupling between the flexural and torsional modes of a microcantilever. The resonance frequency of one mode depends on the amplitude of the other modes. The flexural modes are coupled via the geometric and inertial nonlinearities. The torsional modes exhibit frequency stiffening at high amplitudes, which originates from torsion warping [14]. Interestingly, the nonlinearity constant of one torsional mode changes sign when another torsional mode is driven at high amplitudes. Finally, the coupling between the torsional and flexural modes is studied.

2. Experiment

Microcantilevers are fabricated by photolithographic patterning of a thin low-pressure chemical vapor deposited silicon nitride (SiN) film. Subsequent reactive ion etching transfers the pattern to the SiN layer, and the cantilevers are released using a wet potassium hydroxide etch, resulting in a undercut-free cantilever. The dimensions are length × width × height $(L \times w \times h) = 42 \times 8 \times 0.07\ \mu m^3$. These floppy cantilevers allow high amplitudes and thus facilitate the study of nonlinearities. The cantilever is mounted onto a piezo actuator, which is used to excite the cantilever. The cantilevers are placed in vacuum (pressure $< 10^{-5}$ mbar) to eliminate air-damping and to enable large vibration amplitudes, where nonlinear terms in the equation of motion dominate the dynamics. The cantilever motion is detected using a home-made optical deflection setup which resembles the detection scheme frequently used in scanning probe microscopes. The flexural and torsional vibration modes are detected with a sensitivity of $\pm 1\,\mathrm{pm}/\sqrt{\mathrm{Hz}}$ [15]. A schematic of the measurement setup is shown in Fig. 1(a). The cantilever displacement signal is measured using either a network (NA) or spectrum analyzer (SA). To drive a second mode, a separate RF source is used.

First, the flexural vibrations are characterized by measuring the cantilever frequency response at different resonance modes. The first flexural mode shown in Fig. 1(b) occurs at 54.8 kHz with a Q-factor of 3000. The resonance frequency of the second mode is

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http://dx.doi.org/10.1016/j.ultramic.2012.06.010

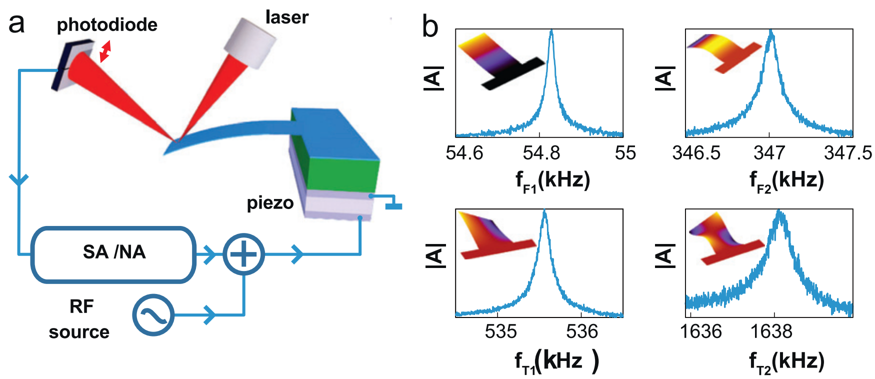

Fig. 1. Measurement setup. (a) Optical deflection setup showing the laser beam, which reflects from the cantilever surface. The spot of the reflected laser beam is modulated in time by a frequency corresponding to the cantilever motion. The cantilever is mounted onto a piezo actuator in vacuum. Network (NA) and spectrum analysis (SA) is performed on the signal from the two-segment photodiode. (b) Frequency responses of the first and second flexural (top panels) and torsional (bottom panels) modes. Inset are the calculated mode shapes from Euler–Bernoulli beam theory.

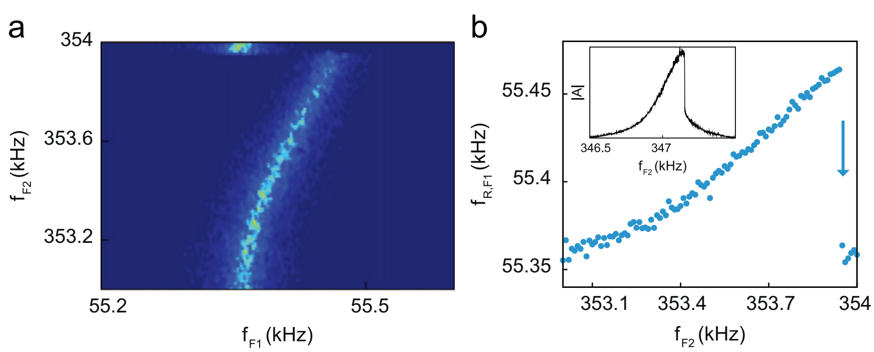

Fig. 2. Flexural–flexural mode interactions. (a) Frequency spectra of the thermal motion of the first flexural mode (f_{F1}), when the second mode is driven through its resonance frequency. Colorscale represents the power spectral density of the displacement noise of mode 1. As the peak width remains constant, there is no significant change in the Q-factor. The motion of the second mode tunes the resonance frequency of the first mode. (b) The resonance frequency of the first mode $f_{R,F1}$ versus the drive frequency of the second flexural mode. The nonlinear response of the second mode is reflected in the fitted resonance frequency of the first mode. Inset: the direct measurement of the nonlinear frequency response of the second mode [20].

347 kHz ($Q=3900$), which is 6.33 times higher than the first resonance mode, in agreement with the calculated ratio $f_{R,F2}/f_{R,F1}=6.27$, following from Euler-Bernoulli beam theory. Not shown is the third flexural mode at 974.9 kHz, with $f_{R,F3}/f_{R,F1}=17.8$, near the expected ratio of 17.6. This indicates that in the linear regime the cantilever beam is described by the Euler-Bernoulli beam theory. Throughout the manuscript, the subscripts *Fi* and *Ti* indicate the frequency span around the *i*th flexural (*F*) or torsional (*T*) resonance mode. The subscript *R* refers to the resonance frequency of that particular mode.

The torsional modes are characterized by rotating the cantilever over 90° in the setup; the two-segment photodiode is then sensitive to vibrations corresponding to torsional resonance modes [15]. The frequency response of the first two torsional modes is shown in Fig. 1(b). From theory, the ratio between the lowest two resonance frequencies of the torsional modes is 3, which is close to the measured ratio of $f_{R,T2}/f_{R,T1}=1638$ kHz$/535.4$ kHz$=3.06$. The Q-factors of the first and second torsional mode are 4300 and 3200 respectively.

At high drive amplitudes, the flexural and torsional modes become nonlinear. The nonlinearity of the flexural modes in a cantilever beam was theoretically studied by Crespo da Silva [16,17]. To include the torsional nonlinearity, the equations of motion are extended (Appendix A). For the flexural and torsional modes, the nonlinearity causes a Duffing-like frequency stiffening when the mode is strongly driven [18,19] leading to a bistable vibration amplitude. This bifurcation is observed in all modes studied in this paper. These nonlinearities are responsible for the coupling between the flexural–flexural, torsional–torsional and flexural–torsional modes.

3. Modal interactions in a microcantilever

We now experimentally demonstrate the coupling between the modes of a microcantilever. We use a two-frequency drive signal to excite two resonance modes of the cantilever simultaneously while we measure the motion of one mode. First, we focus on the interactions between the flexural modes. Then we turn our attention to the torsional modes, starting with the amplitude-dependent resonance frequency of the torsional vibrations, followed by the demonstration of the coupling between the lowest two torsional modes. Finally, the interactions between flexural and torsional modes are discussed.

3.1. Flexural–flexural mode interaction

To investigate the interactions between the two lowest flexural modes, the thermal motion of the first mode is measured with a spectrum analyzer, while the RF source strongly drives the second mode. The thermal noise spectra of the second mode as a function of the drive frequency of the second mode are shown in Fig. 2(a). The color scale represents the power spectral density of the displacement around the resonance frequency of the first mode. A shift of the resonance peak of the first mode is observed as the drive signal at f_{F2} approaches the nonlinear resonance of