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Effective mass and flow patterns of fluids surrounding microcantilevers

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Abstract

An analytical approach to determine the streamlines of fluid flow adjacent to the surfaces of vibrating cantilevers is presented. Fluid flow over the top and bottom surfaces of a microcantilever is established by solving two-dimensional Navier–Stokes equations for viscous flow. The x and y velocity components are used to establish streamlines for absolute fluid motion. These streamlines show a central stagnation core perpendicular and central to the cantilever surface extending along the full length of cantilevers, which most likely accounts for the added mass effect (induced mass) of fluid media around vibrating microcantilevers.

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1. Introduction

Recently microcantilever based sensors have been attracting attention due to their extreme high sensitivity and their potential for an inexpensive, array-based sensing platform. Microcantilever sensors can be operated in a static mode where adsorption of molecules on one of the surfaces, causes the cantilever to bend due to mechanical forces involved in the adsorption process. The extent of bending is proportional to the number of molecules adsorbing on the cantilever surface [1]. Simultaneously, cantilevers can also be operated in a dynamic mode where the resonance frequency of the cantilever varies as a function of mass loading. The cantilever can be excited into resonance by mechanically exciting the cantilever with a number of different techniques. Since the cantilevers are extremely small they also execute thermal motion (Brownian motion) where the cantilever in thermal equilibrium with ambient acts like a giant molecule or a colloidal particle. The amplitude of the thermal motion, however, is extremely small. Mass loading on the cantilevers due to molecular adsorption can be measured with extreme high sensitivity when the cantilever resonance frequency is very high. Therefore, cantilevers with small mass and high spring constant are ideal for achieving high mass sensitivity due to molecular adsorption. Since cantilevers can be designed to have extremely high frequency, frequency shift based approach has the potential of being more sensitive than stress based approach where sensitivity is higher for small spring constant, low frequency cantilevers. Small spring constant, however, increases the noise in the system induced by thermal ambient.

It has been commonly observed that the density of fluid media surrounding vibrating microcantilevers affects frequency response. For example, when the environment changes, say, from helium to air to argon, the frequency at peak amplitude reduces as shown in Fig. 1. The shift in frequency due to the medium is attributed to change in effective mass of the cantilever. The effective mass is a sum of the cantilever mass and an induced mass on the cantilever. The induced mass is related to a mass of fluid undergoing acceleration along with the vibrating cantilever. Because the density of helium is the lightest, we see a reduction in frequency as the environment changes to air and then to argon. The shape (related to Q factor) of the

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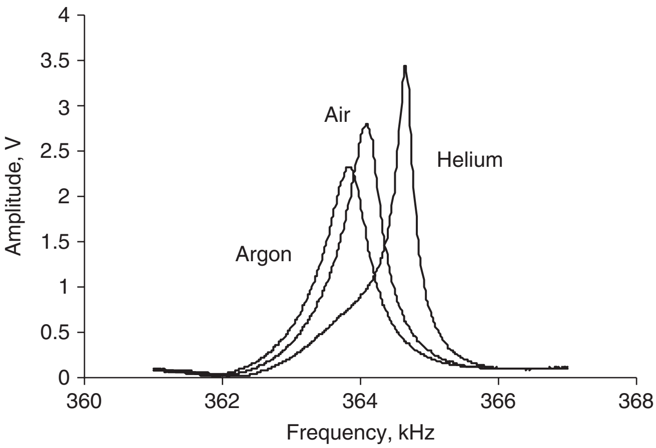

Fig. 1. Frequency responses of a microcantilever in helium, air and argon (Oak Ridge National Laboratory).

frequency response curve is an indication of the magnitude of the damping factor. Argon produces the highest damping factor. Helium produces the lowest. Since the elastic stiffness is unchanged for both cases, the frequency shift has been explained as a change in induced mass and/or a change in vibration dampening. While both cause a shift in the frequency at peak amplitude, vibration dampening accounts for only a slight amount of the frequency shift. Induced mass has a far greater effect. Induced mass changes natural frequency. Damping does not change natural frequency but does bring about a slight shift in the frequency at peak vibration amplitude.

Sader [2] couples hydrodynamics with cantilever motion in a comprehensive mathematical analysis to show how effective mass and viscosity can be extracted from frequency response data. His hydrodynamic solution for a rectangular beam is based on the work of Tuck [3], who showed that the hydrodynamics for a circular cylinder and a rectangular beam are similar as the Reynolds number approaches zero. Theoretical predictions of viscosity and density, using this procedure, compare favorably with documented data [4]. Kirstein [5] predicts fluid flow by approximating the rectangular cross section with an elliptical geometry, based on Lambs [6] classic solution for viscous flow around cylinders/ellipses. Shih [7] models fluid flow by using Stokes [8] classic solution for fluid flow around a sphere.

The purpose of our paper is to graphically show fluid flow streamlines and use them to indicate how effective mass is generated.

2. Mathematics of Fluid Flow

A key assumption in the mathematical development is that fluid flow around microcantilevers is laminar. The geometry of streamlines is based on the direction of local x and y velocity components, which is used to incrementally trace the path of the fluid numerically. These two velocity components, $u(x,y)$ and $v(x,y)$, are determined by solving Navier-Stokes equations (Eq. 1 and 2). The mass term in both equations has been omitted for the sake of simplicity. Our studies indicate that Eqs. 1 and 2 are adequate for predicting streamlines for gaseous environments. The streamlines for liquids are similar but distorted somewhat due to the addition of the linear acceleration terms.

The coordinate system used in the development is illustrated in Fig. 2. A typical surface section of length, Δz, and width, $2a$ is taken as the base of the control volume shown in Fig. 3. The pressure function, $p(x,y)$, and velocity functions, $u(x,y)$ and $v(x,y)$ vary with x and y and are assumed constant over the unit distance, Δz. Note that $u(x,y)$ is the local velocity component in the x direction, while $v(x,y)$ is the local velocity component in the y direction.

The control volume under consideration is defined by a rectangle having dimensions of $2a$ x h x Δz (see Fig. 3). Fluid flows into and out of this control volume to maintain continuity of flow. Fluid is assumed to be incompressible.

Based on the above assumptions, the two-dimensional Navier–Stokes equations simplify to [9]

$$\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{1}$$

$$\frac{\partial p}{\partial y} = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right). \tag{2}$$

These equations, plus the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

establish fluid flow patterns within the control volume.

The simplified Navier–Stokes equations and continuity equation can be combined to produce the Laplace equation (Eq. (4)) with pressure, $p(x,y)$, as the dependent variable.

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0. \tag{4}$$

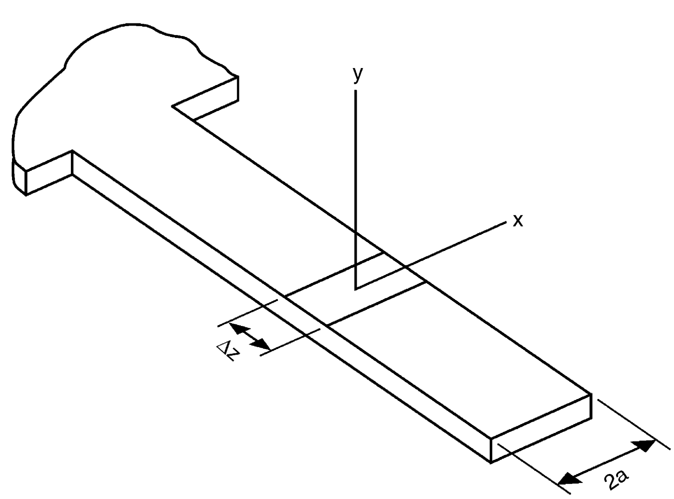

Fig. 2. Microcantilever with coordinate system.