Chinese Journal of Chemical Engineering 24 (2016) 737–748

Contents lists available at ScienceDirect

Chinese Journal of Chemical Engineering

journal homepage: www.elsevier.com/locate/CJChE

Fluid Dynamics and Transport Phenomena

Analysis of the nonlinear dynamic characteristics of two-phase flow based on an improved matrix pencil method☆

Hongwei Li *, Junpeng Liu, Yunlong Zhou, Bin Sun

School of Energy and Power Engineering, Northeast Dianli University, Jilin 132012, China

A R T I C L E   I N F O

Article history:
Received 15 June 2015
Received in revised form 2 September 2015
Accepted 3 October 2015
Available online 22 January 2016

Keywords:
Matrix pencil (MP) method
Component energy index (CEI)
Stability diagram
Flow pattern identification
Flow dynamics

A B S T R A C T

Gas–liquid two-phase flow is complex and has uncertainty in phase interfaces, which make the two-phase flow look very complicated. Even though the flow behavior (*e.g.* coalescence, crushing and separation) of single bubble or bubble groups in the liquid phase looks random, combining some established characteristics and methodologies can find regularities among the randomness. In order to excavate the nonlinear dynamic characteristics of gas–liquid two-phase flow, the authors developed an improved matrix pencil (IMP) method to analyze the pressure difference signals of the two-phase flow. This paper elucidates the influence of signal length on MP calculation results and the anti-noise-interference ability of the MP method. An IMP algorithm was applied to the fluctuation signals of gas–liquid two-phase flow to extract the mode frequency and damping ratio, which were combined with the component energy index (CEI) entropy to identify the different flow patterns. It is also found that frequency, damping ratio, CEI entropy and stability diagram together not only identify flow patterns, but also provide a new way to examine and understand the evolution mechanism of physical dynamics embedded in flow patterns. Combining these characteristics and methods, the evolution of the nonlinear dynamic physical behavior of gas bubbles is revealed.

1. Introduction

Gas–liquid two-phase flow in narrow rectangular channels has been the subject of increased research interest in the past decades [1]. This situation exists commonly in power engineering, petrochemical industries, and heat/mass transfer operations, also in chemical engineering equipment, such as evaporators, condensers, reactors, and heat exchangers. In gas–liquid two-phase flow, the flow pattern is the basis of successive heat and mass transfer research. Different flow patterns have different impact on heat transfer, *i.e.*, the appearance of slug flow and mist flow can cause heat transfer deterioration, which is very dangerous in high temperature and high pressure conditions. All these high-risk heat transfer operations require accurate identification of two-phase flow patterns. Safe and responsible development of methodologies associated with reliable heat transfer performance requires the understanding of dynamic physical characteristics for two-phase flow for better industrial system optimization design and dynamic monitoring of working conditions.

Research on conventional-sized channel flow pattern already has a good foundation [1,2]. Since the1990s, a trend has been growing in the study of flow pattern identification, based on a method of chaos detection, fractal analysis, and complex networks as well as time and frequency domain analysis [3]. Jin *et al.* [4–6] performed dynamics analysis on oil and gas–liquid two-phase flow conductance signals by using a multiple chaotic parameter index; this analysis was limited by the center of multiple gravity trajectories in the phase space. Progress has been made in revealing the mechanism of two-phase flow patterns. Gao and Jin [7] applied the complex network on the mechanical characteristics of two-phase flow to evolution analysis, and revealed the detailed mechanism of interaction between gas and liquid. Sun. *et al.* [8] provided the identification and analysis for the gas–liquid two-phase flow pattern and flow characteristics of a horizontal Venturi tube by adapting the optimal kernel time–frequency analysis method. Du *et al.* [9] analyzed the conductance signals of the gas–liquid two-phase flow by using the optimal kernel time–frequency characterization method. They

☆ Supported by the National Natural Science Foundation of China (51406031), Jilin City Science and Technology Plan Project (201464055) and Jilin Province Education Department Science Research Project (2015-243).

* Corresponding author.
E-mail address: lihongweihw@163.com (H. Li).

http://dx.doi.org/10.1016/j.cjche.2016.01.015
1004-9541/

accurately distinguished the different flow patterns by extracting the two time–frequency characteristic values (total energy and entropy). Manfredo [10] characterized flow properties by the phase density time–frequency distribution characteristics of gas–liquid two-phase flow image signals. Ommen *et al.* [11] reviewed two-phase time–frequency domain analysis methods and analyzed their advantages/disadvantages in the characterization of two-phase flow dynamics characteristics. The method extracting the vibration mode and their frequency and damping ratio have been applied to the two-phase flow field. This kind of algorithm mainly includes wavelet transform [12], Hilbert–Huang transform (HHT) [13,14] and stochastic subspace identification (SSI) [15].

Compared to regular channels, the information collected from tiny channels is more sensitive to changes in experimental environment, and noise reduces more severely the amount of effective information. Therefore, revealing the inherent nonlinear dynamic characteristics of a tiny channel as easily as that in a regular channel seems difficult. In this case, an effective algorithm is required to prevent noise and acquire the essentials from signals.

Matrix pencil (MP) [16,17] is an algorithm of vibration mode parameter identification, which can obtain the vibration mode of different orders. Hence, the main frequency and damping ratio can be calculated from different vibration modes. This method has been applied in power systems and in the construction field [18]. However, the traditional matrix pencil algorithm is not good at identifying the mode order of signals under noisy conditions. This paper uses the stability diagram [19] to set the order of signals, and applies component energy index (CEI) to distinguish the false modals. After processing the signal order and CEI intervention by the improved MP (IMP) method, we can obtain the main vibration mode that represents typical characteristic of temporal signal. The IMP method can improve the accuracy of mode parameter identification and reduce the calculation load. This paper presents the best application of IMP method and examines the influence of signal length on the IMP calculation results and the anti-noise-interference ability. Finally, the IMP method was used to analyze the differential pressure signals of gas–liquid two-phase flow in a narrow rectangular vertical upward pipe (the inner diameter is 2.0 mm × 0.81 mm). The main vibration mode frequency, damping ratio and CEI entropy were extracted to identify the flow patterns. The evolution of nonlinear dynamic physical behavior of gas bubbles was also presented.

2. Theoretical Basis

2.1. Matrix pencil

2.1.1. Matrix pencil theory

The phase interaction in gas–liquid two-phase flows is featured with high speed and uncertainty distribution characteristics. The two-phase flow fluctuation signals present coupling oscillation characteristics of different oscillation modes. The whole system can be assumed as the linear superposition, wherein the mode can be fit to

$$y(kT_s) = \sum_{i=1}^{M} R_i \mathrm{e}^{(-\alpha_i + jw_i)kT_s} \tag{1}$$

where $k = 0,1,...,N\text{-}1$, N is the number of sampling points; R_i, α_i, w_i is the complex amplitude, attenuation factor and angular frequency of the ith vibration mode; M is the signal order, and T_s is the sampling interval.

Using the sampling signal $\boldsymbol{y} = [y(0), y(1), \cdots, y(N-1)]^{\mathrm{T}}$ to structure the Hankel matrix $\boldsymbol{Y}_{N-L\times(L+1)}$, normally, $L = N/4$ - $N/3$. The singular value decomposition can be used on the $\boldsymbol{Y}$ matrix [20]:

$$\boldsymbol{Y} = \boldsymbol{USV}^{\mathrm{T}} \tag{2}$$

Taking columns which have the former M larger singular values σ_i of diagonal matrix $\boldsymbol{S}$ to make up the matrix $\boldsymbol{S}_1$, we were able to create the right singular vectors v_i corresponding to σ_i which formed $\boldsymbol{V}' = [v_1, v_2, \ldots, v_M]$. The matrix $\boldsymbol{V}_1$ is constructed by the matrix $\boldsymbol{V}'$ without the first line, and the matrix $\boldsymbol{V}_2$ is constructed by $\boldsymbol{V}'$ without the last line.

Defining $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$ as follows:

$$\begin{cases} \boldsymbol{Y}_1 = \boldsymbol{US}_1\boldsymbol{V}_1^{\mathrm{T}} \\ \boldsymbol{Y}_2 = \boldsymbol{US}_1\boldsymbol{V}_2^{\mathrm{T}} \end{cases} \tag{3}$$

Making $z_i = \mathrm{e}^{(-\alpha_i + jw_i)k}$, we can prove that z_i is the generalized eigenvalue of matrix pencil $\boldsymbol{Y}_2 - \lambda\boldsymbol{Y}_1$, namely by solving

$$\boldsymbol{G} = \boldsymbol{Y}_1^{+}\boldsymbol{Y}_2 \tag{4}$$

where $\boldsymbol{Y}_1^{+}$ is the pseudo-inverse matrix of $\boldsymbol{Y}_1$. $\boldsymbol{G}$ has n eigenvalues denoted by $z_i (i = 1, 2, 3, \ldots, n)$.

After determining the characteristic value z_i of matrix $\boldsymbol{G}$, the frequency and damping ratio can be obtained by

$$f_i = \frac{1}{2\pi}\mathrm{Im}\left(\frac{\ln z_i}{T_s}\right) \tag{5}$$

$$\sigma_i = Re\left(\frac{\ln z_i}{T_s}\right) \tag{6}$$

2.1.2. Evaluation methods

Noise resistance was regarded as an index in this study to evaluate the method of dealing with a time series with strong noise pollution. The influence of calculation results by signal length was also considered in this section. Three kinds of simulation signals: chaos (Lorenz), fractal (Conway) and periodicity (Sine) time series were employed as examples for this purpose. The reason for selecting this sequence as the simulation signals is that the signals collected from experiments usually have typical chaotic, fractal and periodicity characteristics. Another reason is that these simulation time series exhibit typical dynamic behavior, being both typical and classic.

The Lorenz, Conway and Sine series contain different types of noise composed of several groups of time series produced under the following conditions.

(1) Lorenz chaotic signal by the Lorenz equation:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= -10(x-y), \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= -y + 28x - xz, \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= xy - \frac{8}{3}z, \end{aligned} \tag{7}$$

with the initial condition $x = 2$, $y = 2$, $z = 20$, the iteration is by fourth-order Runge–Kutta method, and the variable $\boldsymbol{X}$ taken as the simulation sequence (as shown in Fig. 1(a), S1) [21]. Two kinds of noise signal are: noise1 = awgn(x,40 dB) (variable x is mixed with the SNR of 40 dB Gaussian white noise, as shown in Fig. 1(a), S2), noise2 = x + 0.2 × wgn(40 dB) (variable x is directly mixed with the noise intensity of 40 dB Gaussian white noise, as shown in Fig. 1(a), S3). The same noise adding method is applied onto the following two signals.

(2) Conway fractal signal & noise-added signals. Conway sequences are the fractal time series (as shown in Fig. 1(b),