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Design optimization using Statistical Confidence Boundaries of response surfaces: Application to robust design of a biomedical implant

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ABSTRACT

This paper deals with the use of Statistical Confidence Boundaries (SCB) of response surfaces in robust design optimization. An empirical model is therefore selected to describe a real design constraint function. This constraint is thus approximated by a second order polynomial expansion which is fitted to numerical simulations that use a Finite Element Method (FEM). A technique is also proposed to analyze the effects of the uncertainties of the inputs of the simulations. This approach is employed to optimize the design of a biomedical wrist implant. A real optimized implant is then manufactured and tested to validate the numerical model.

1. Introduction

In the common life, wounds can appear, due to sports activities or accidents in the home. Often, prostheses or implants are then used to restore the functional capacity of the patient [1]. The design of biomedical products is however constrained by many criteria (biocompatibility of the materials, lifespan of the implant, small available geometrical space. . .) and needs multi-objective optimization methods. An overview of Design Theories and Methodologies (DTM) was already presented in a keynote paper [2]. The prescriptive models for design were thus discussed (canonical design process, morphological analysis and prescriptive models of the design artefacts, Suh's Axiomatic Design and Taguchi Method). These DTM methods can be used to enrich the functional and attributive information of design solutions. Different approaches are then employed to characterize the Design Solution Surface (DSS). The proposed models can thus be classified with two different points of view: Global/local description and Analytical/Numerical modelling. As example, the optimization of the deflection of a cantilever is presented in Fig. 1. The first approach allows calculating the design solution for any set of input parameters. The quality of the results also only depends on the accuracy of the model. Generally, the time and cost to develop a global analytical model are however significant. For that reason, empirical models are employed in the second approach to describe the real design constraint function. Usually, a polynomial model is then locally best fitted to the real DSS, using a limited number of numerical simulations or real experiments. A design of experiment (DOE) technique can therefore be employed to define the optimal set of input parameters of this approach. Optimization of the design parameters is however limited to the local domain used for fitting. In the third approach a single numerical simulation is carried out, just to check that the design constraints are satisfied for the given selected set of inputs. This simulation is usually based on a Finite Element Model (FEM). In the last approach, the numerical simulation is repeated many times to get a global view of the DSS. This method, however, only provides a discrete description of the DSS. In the second approach, the quality of the best fit is a central property for the accuracy of the optimization. In some works [3,4], the major components of Engineering Design Optimization (EDO) were classified in five entities: design variables, constraints, objective functions, problem domain and environment. Their uncertainties or variations are propagated to the optimized design solution to check the robustness of the design [5]. This permits also verifying that the design requirements will be satisfied for all manufactured products. Robust design optimization [6–8] can be used to design biomedical products. Roy et al. conclude their keynote paper with this sentence: "*there is a lack of research in multi-objective design optimization that deals with*

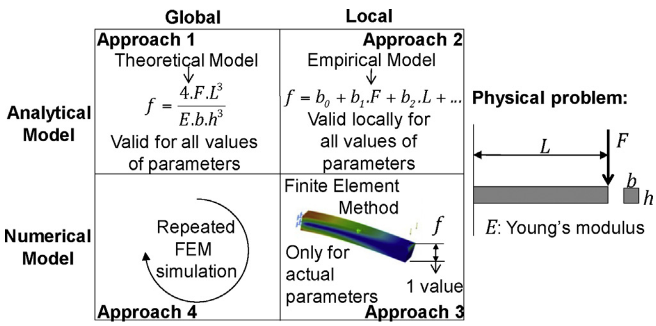

Fig. 1. Different approaches to characterize design solution surfaces.

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uncertainty and constraints together" [3]. In this context, the aim of our paper is to use the Statistical Confidence Boundary (SCB) of response surfaces in design of experiment-based optimization.

2. Presentation of the method

This section presents the methodology used to get the response surface and the related SCBs, of a DSS characterized by FEM simulations. As example, the approach will focus on a problem with two factors (*U1* and *U2*). The response surface will be approximated by the second order polynomial empirical model described by Eq. (1).

$$y = \sum_{i=0}^{2}\sum_{j=i}^{2} b_{ij} \cdot x_i \cdot x_j \quad (1)$$

where $x_0 = 1$. x_1, x_2 are the normalized values of factors *U1*, *U2*.

The different possible shapes of response surfaces are drawn in Fig. 2. A DOE strategy permits defining the optimal set of inputs (*U1*, *U2*) to be employed for the FEM simulations. The coefficients of the polynomial model are then derived from the FEM results using the following pseudo-inverse calculation:

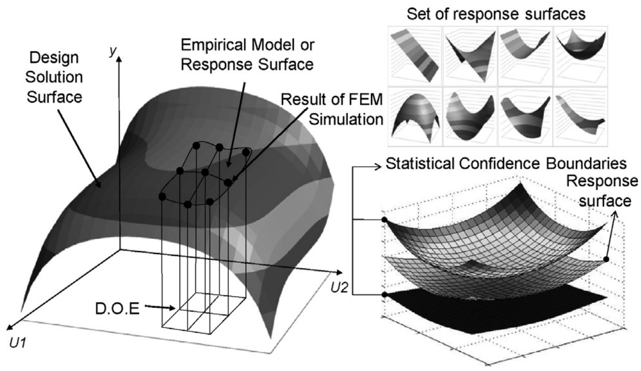

Fig. 2. Principle of D.O.E based optimization.

$$\begin{aligned} &\boldsymbol{Y} = \boldsymbol{X} \cdot \boldsymbol{B} \text{ with } \boldsymbol{X} \text{ the matrix of the normalized products } x_i \cdot x_j \\ &\qquad \text{and } \boldsymbol{B} \text{ : the response surface coefficients vector} \\ &\hat{\boldsymbol{B}} = (\boldsymbol{X}^T \cdot \boldsymbol{X})^{-1} \cdot \boldsymbol{X}^T \cdot \boldsymbol{Y} \end{aligned} \quad (2)$$

$\hat{\boldsymbol{B}}$ defines the best estimate of the surface response coefficients.

A propagation method is then implemented to account for the uncertainties of the input parameters and the inaccuracy of the model. The scatter of the material properties, design parameters and manufacturing conditions is assumed random and normal distributed. It is described by standard deviations (σ).

The inaccuracy of the fitting model is characterized by the root mean square (Rms) of the differences between the FEM simulation results and the mean local polynomial used to describe the DSS. Variations of the environment are not taken into account in this study. The scheme used to propagate the different deviations to the SCB is presented in Fig. 3.

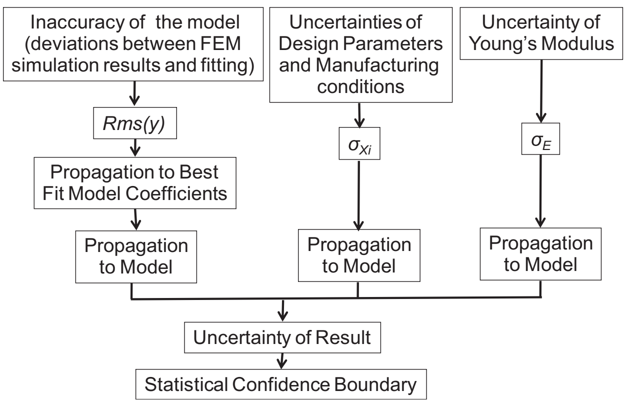

Fig. 3. Uncertainty and Error Propagation scheme.

2.1. Propagation of the inaccuracies of the fitting model

The inaccuracy of the model is represented by the root mean square Rms(y) of the best fit residues **R** calculated through Eq. (3).

$$\boldsymbol{R} = \boldsymbol{Y} - \boldsymbol{X} \cdot \hat{\boldsymbol{B}} \quad (3)$$

The mean square error matrix $\mathbf{MSE}(\hat{\boldsymbol{B}})$ of the response surface coefficients is then calculated using following expression:

$$\begin{aligned} &\mathbf{MSE}(\hat{\boldsymbol{B}}) = \mathbf{MSE}((\boldsymbol{X}^T \cdot \boldsymbol{X})^{-1} \cdot \boldsymbol{X}^T \cdot \boldsymbol{Y}) \\ &\mathbf{MSE}(\hat{\boldsymbol{B}}) = (\boldsymbol{X}^T \cdot \boldsymbol{X})^{-1} \cdot \mathrm{Mse}(y) = (\boldsymbol{X}^T \cdot \boldsymbol{X})^{-1} \cdot \mathrm{Rms}(y)^2 \end{aligned} \quad (4)$$

This permits finally evaluating the mean square error Mse($\hat{y}$) of the design solution $\hat{y}$ estimated for any set of input parameters. Following classical propagation method is used for that purpose:

$$\mathrm{Mse}(\hat{y}) = \boldsymbol{J}_{\hat{\boldsymbol{B}}} \cdot \mathbf{MSE}(\hat{\boldsymbol{B}}) \cdot \boldsymbol{J}_{\hat{\boldsymbol{B}}}^T \quad (5)$$

with, $\boldsymbol{J}_{\hat{\boldsymbol{B}}}$, Jacobian of function $\hat{y}$ with respect to coefficients b_{ij}.

2.2. Propagation of the uncertainties of the design parameters

The scatter of the design parameters is usually characterized by Tolerance Intervals (*TI*) that are either imposed by the designer or derived from the capability of the manufacturing process. If a Gaussian Probability Density Function (PDF) is assumed for a given input parameter x_i, the standard deviation σ_{xi} can be derived from the tolerance interval TI_i through following expression:

$$\sigma_{xi} = \frac{TI_i}{6} \quad (6)$$

Assuming the independence of the two parameters x_1, x_2, the standard deviations are then propagated to the response surface, using Eq. (7).

$$\mathrm{Var}(\hat{y}) = \boldsymbol{J}_{xi} \cdot \mathbf{VAR}(x_i) \cdot \boldsymbol{J}_{xi}^T \quad (7)$$

with, $\boldsymbol{J}_{xi}$, Jacobian of function $\hat{y}$ with respect to x_i

$$\mathbf{VAR}(x_i) = \begin{pmatrix} TI_1^2/36 & 0 \\ 0 & TI_2^2/36 \end{pmatrix} \text{ is the covariance matrix of } x_i.$$

2.3. Propagation to load and stress, of the scatter of the material properties, in the case of a pure elastic behaviour

Mechanical tests (tensile tests) are usually carried out to characterize the material properties. They allow defining the mean Young's modulus E of the material, and the related standard deviation σ_E. Common constrains imposed in design optimization are the maximum stress (S) or applied load (L) to which the structure must resist. In the case of imposed displacements, and pure elastic behaviour of the material, the resulting applied Load and stress are proportional to Young's modulus. This leads to following relationships:

$$L = k_L \cdot E \Rightarrow \mathrm{Var}(L) = k_L^2 \cdot \mathrm{Var}(E) \Rightarrow \boldsymbol{\sigma}_L^2 = (L/E)^2 \cdot \sigma_E^2 \quad (8)$$

$$S = k_S \cdot E \Rightarrow \mathrm{Var}(S) = k_S^2 \cdot \mathrm{Var}(E) \Rightarrow \boldsymbol{\sigma}_S^2 = (S/E)^2 \cdot \sigma_E^2 \quad (9)$$

2.4. Response surface and Statistical Confidence Boundary

Previous calculations are used to evaluate the design solution $\hat{y}$ for any set of input parameters and estimate its mean square error (Eq. (10)).

$$\mathrm{Mse}(\hat{y}) = \boldsymbol{J}_{\hat{\boldsymbol{B}}} \cdot \mathbf{MSE}(\hat{\boldsymbol{B}}) \cdot \boldsymbol{J}_{\hat{\boldsymbol{B}}}^T + \boldsymbol{J}_{Xi} \cdot \mathbf{VAR}(X_i) \cdot \boldsymbol{J}_{Xi}^T + \left(\frac{L \text{ or } S}{E}\right)^2 \cdot \mathrm{Var}(E) \quad (10)$$

With smooth design solution surfaces, the model inaccuracy remains small in comparison to the random perturbations of the inputs that are assumed to be normal distributed. The global distribution is therefore close to a Gaussian. The SCBs of