CIRP Annals - Manufacturing Technology 63 (2014) 353–356

Contents lists available at ScienceDirect

CIRP Annals - Manufacturing Technology

journal homepage: http://ees.elsevier.com/cirp/default.asp

Analytical prediction of contact stiffness and friction damping in bolted connection

Eiji Shamoto (1)[a,*], Yohei Hashimoto[a], Miki Shinagawa[b], Burak Sencer[a]

[a] Department of Mechanical Engineering, Nagoya University, Chikusa-ku, Nagoya, Aichi, Japan
[b] IHI Corporation, Shin-Nakahara-cho, Isogo-ku, Yokohama, Japan

ARTICLE INFO

Keywords:
Structural analysis
Damping
Contact stiffness

ABSTRACT

This paper presents an analytical method to predict contact stiffness and friction damping in bolted connections. Despite its importance in machine design, analytical prediction of the contact parameters has not been realized because of difficulty in time-history nonlinear analysis of sticking/sliding contact. In this study, a torsional contact model around a connecting bolt is developed. Linear phenomena are extracted and solved by FEM analysis, and then linear combination of the FEM results is calculated iteratively to search sticking/sliding zones in torque equilibrium. Proposed model is experimentally validated, and it will lead to fully analytical modelling of practical machines.

1. Introduction

Structural analysis is a powerful tool to predict static and dynamic behaviour of machine tools including deformations, stress distribution, vibration modes and resonant frequencies. The analysis requires several parameters such as structural dimensions and material properties, i.e. density, elastic modulus and Poisson's ratio. These parameters can be known with moderate accuracy in the design stage, except the non-linear contact stiffness and the friction damping in fixed and sliding connections between the components. These two parameters have been the bottleneck to achieve accurate analytical prediction of static and dynamic behaviour of multi-component machine structures. Especially, prediction of the non-linear damping coefficient is foremost important due to the fact that damping in many machine structures is mainly caused by the friction in connections where the others such as material damping in continuous media and the viscous damping due to ambient air are much smaller in many cases. Nevertheless, prediction of non-linear friction may require impractical computation time due to complex statically-indeterminate history-dependent non-linear nature of the problem.

As a result, only a limited amount of research is directed to analytical prediction of contact stiffness and damping coefficients making experimentally identified or empirical parameters to be input in conventional structural analysis [1]. There have been some attempts to develop quasi-analytical models, but these cannot be generalized as they contain experimentally identified coefficients like the "slip ratio" or "slip displacement" [2,3]. An exception is the prediction of normal contact stiffness, which can be solved by the help of Hertzian contact theory in special cases [4]. Similarly, authors have developed analytical methods to predict friction damping in a sliding table [5] and friction loss energy in torsional vibration between bolted plates [6].

This paper presents an analytical method to predict the contact stiffness and the friction damping in a bolted connection. To the best of the authors' knowledge, this is the first attempt of fully analytical prediction of contact stiffness and damping coefficient of mechanical structures with bolted connections. The sticking/sliding contact is a complex non-linear problem, which occurs in bolted connections and requires impractical computation time. An efficient calculation algorithm is developed in this study. The linear phenomena in the contact stress distribution and torsional deformation are extracted and analyzed preliminarily by FEM. The non-linear contact problem is then iteratively solved by simple linear combination of the FEM results, making the complex computation possible in practice. The proposed method is validated on an instrumented mechanical structure containing bolted connections. The proposed method predicts the contact stiffness accurately and the damping coefficient in bolted connections utilizing only the structural dimensions, bolt clamping force, material properties and the friction coefficient between the bolted components.

2. Proposed model of bolted connection

2.1. Outline of the contact model

In this study we consider a bolted connection, where two mechanical parts are connected with one bolt, and a torsional displacement or vibration is applied between those parts as shown in Fig. 1. The non-linear sticking/sliding phenomena are simulated as the torsional displacement Ω is increased from 0 to the maximum value Ω^{amp} to predict the contact stiffness and friction damping in a bolted connection. The contact is analyzed in 3 regions depending on the amplitude of the vibration. When the plates are large enough, the contact stress σ becomes zero in outer region of the connection. This region is called the "non-contact zone" in this study. When torsional displacement is applied, micro slip occurs in the outer region of the inner contact zone. This region

* Corresponding author.

http://dx.doi.org/10.1016/j.cirp.2014.03.134
0007-8506/© 2014 CIRP.

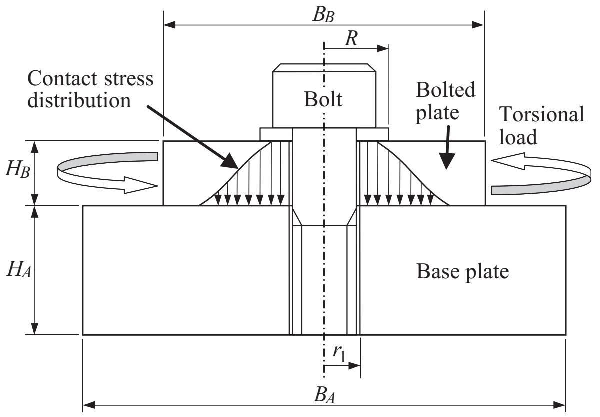

Fig. 1. Bolted plate and base plate with torsional load.

is called the "sliding zone". Finally the plates stick in the inner region of the contact and this region is defined as the "sticking zone". As the displacement increases, the sliding zone expands to the inside and the sticking zone reduces. This reduced sticking zone leads to smaller contact stiffness, and the increased sliding zone generates larger friction loss, i.e. larger friction damping. If the torque exceeds a certain limit, the sticking zone disappears and the plates slide in the whole contact zone.

This sticking/sliding connection is modelled as shown in Fig. 2. The centre of the connecting bolt is located at the origin of the Cartesian (r_x and r_y) coordinate system. The contact stress σ due to the bolt clamping force P is distributed on the contact surface as shown by the orange line in Fig. 2. The bottom surface of the base plate is fixed, and the torsional displacement is applied to the outer upper edge of the bolted plate. r_Ω coordinate shows the applied torsional displacement of the outer upper edge of the bolted plate Ω_i at the ith discrete step ($i = 1, 2, \ldots, n$). Φ_i is the torsional displacement of the sticking surface of the base plate in the sticking zone, while Θ_i ($= \Omega_i - \Phi_i$) is torsional displacement between the sticking surface and the outer upper edge of the bolted plate. $\phi_{i,j}$ is torsional displacement of the upper surface of the base plate at the jth radius $r_j = j \cdot \Delta r$ ($j = 1, 2, \ldots, m$). $\theta_{i,j}$ is torsional displacement between the lower surface and the outer upper edge of the bolted plate.

In the sticking zone, $j \leq stk$, the upper surface of the base plate and the lower surface of the bolted plate move together. In the sliding zone, $stk < j \leq sld$, the two plates slide with frictional force q_j.

$$q_j = \mu \cdot \sigma_j \cdot \pi \left\{ \left(r_j + \frac{\Delta r}{2} \right)^2 - \left(r_j - \frac{\Delta r}{2} \right)^2 \right\} = \mu \cdot \sigma_j \cdot 2\pi r_j \Delta r \quad (1)$$

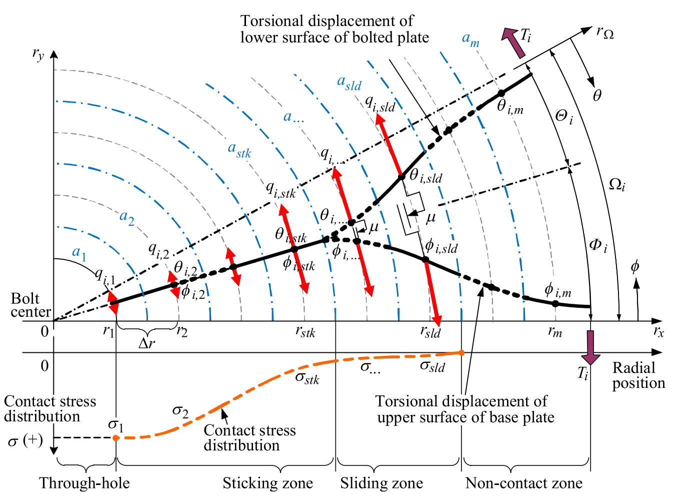

Fig. 2. Model of contact stiffness/friction damping at bolted connection with torsional load.

where $2\pi r_j \Delta r$ is area of the jth small circular ring. μ is friction coefficient between the plates, which is assumed to be 0.17 based on measured frictional force at various speed and normal load [6].

Sliding distance $u_{i,j}$ is given as follows.

$$u_{i,j} = r_j \cdot (\Omega_i - \psi_{i,j}) = r_j \cdot (\Omega_i - \phi_{i,j} - \theta_{i,j}) \quad (2)$$

where combined displacement $\psi_{i,j}$ represents sum of the torsional displacements of the two plates, i.e. $\psi_{i,j} = \phi_{i,j} + \theta_{i,j}$.

Small friction loss energy $w_{i,j}$ at the jth radial interval is derived by multiplying the above frictional force and distance, and the total energy W_i, dissipated until the displacement reaches Ω_i, can be obtained by summing up the small energy in the sliding zone.

$$W_i = \sum_{j=1}^{m} w_{i,j} = \sum_{j=stk+1}^{sld} w_{i,j} = \sum_{j=stk+1}^{sld} q_{i,j} \cdot u_{i,j} \quad (3)$$

By equating total dissipation energy over one vibration cycle, equivalent damping coefficient D_e can be expressed as follows:

$$D_e = \frac{W}{\pi \omega (\Omega^{amp})^2} \quad (4)$$

where ω is angular frequency, and Ω^{amp} is angular amplitude.

2.2. Efficient calculation algorithm

The sticking/sliding boundary is searched and the non-linear contact stiffness and damping coefficient are predicted by the following efficient algorithm.

For higher efficiency, the friction loss energy over one vibration cycle W is obtained by 4 times the energy dissipated over the first quarter-cycle, where initial displacements are set to be zero without any sticking/sliding history. This is because half energy $W/2$ is dissipated in every half-cycle by sliding from one extreme displacement to the other, while in the first quarter-cycle $W/4$ is dissipated by sliding from neutral or zero displacement to one extreme displacement. Thus, the torsional displacement is simply increased step by step as follows.

$$\Omega_i = i \cdot \Delta\Omega \quad (5)$$

where i is increased from 1 to n, and $n \cdot \Delta\Omega$ corresponds to the maximum value of the vibration amplitude Ω^{amp}.

A unit bolt-clamping force $P = 1$ [N] is assumed to be equally distributed over the clamped surface whose outer and inner radii are $R = 10$ mm and $r_1 = 5$ mm, and the contact stress distribution is calculated only once by FEM. The result is shown in Fig. 3. It shows that the radius at the contact/non-contact boundary r_{sld} is 21.5 mm. Since this elastic compression is a linear problem, contact stress distribution due to arbitrary bolt-clamping force can be obtained by multiplying the contact stress at every radial position by the force.

The torsional elastic deformation is also a linear problem. However, the sticking/sliding boundary changes depending on clamping force and applied displacement. Hence, the following database is prepared before the non-linear contact analysis is

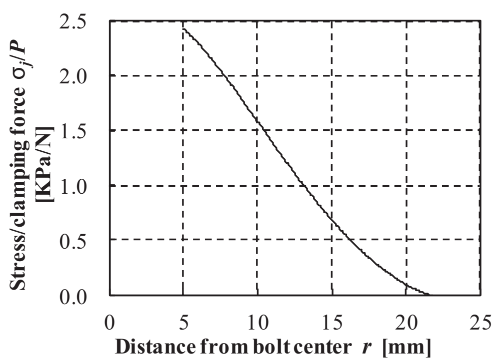

Fig. 3. Distribution of contact stress per unit clamping force calculated by FEM.