Nuclear Instruments and Methods in Physics Research B 356–357 (2015) 8–11

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research B

journal homepage: www.elsevier.com/locate/nimb

Infrared and terahertz radiation of a crystal at axial channeling

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ARTICLE INFO

Article history:
Received 25 March 2015
Accepted 19 April 2015
Available online 14 May 2015

Keywords:
Crystal
Charged particle
Channeling
Infrared radiation
Polarization

ABSTRACT

Basic properties of radiation of a crystal lattice excited by an axial channeling particle are considered. It is shown that a coherent radiation of atoms occurs if the frequency of oscillations of the channeled particle comes to a resonance with the vibrational mode of the crystal. Spectral and angular distribution of radiation and its polarization are calculated. In case of a relativistic channeled particle, the radiation of atoms is generated into a narrow cone in the direction of a crystallographic axis along which the particle is channeling. The radiation of atoms exited at axial channelling has significant degree of circular polarization.

1. Introduction

We investigate a new kind of radiation that occurs at the axial channeling of a charged particle in a crystal lattice. When channeling around the crystalline axis, the charged particle transmits part of its transverse momentum to the atoms of crystalline axis. This excites vibration of the atoms in the axis which are therefore involved in the collective motion. Oscillations of a nucleus with interior electrons cause electromagnetic radiation. This phenomenon is similar to the vibrational excitation of molecules and respective radiation. The phases of atomic vibrations are correlated because the vibrations are excited by the same channeled particle. Therefore, the radiation produced is coherent.

Obviously, the considered radiation competes with radiation caused by thermal oscillations of atoms in the lattice. However, the fundamental difference from the thermal radiation is that the considered oscillations are coherent, hence, the radiation is amplified in certain directions and at distinctive frequencies. Properties of this type of radiation at the planar channeling of relativistic particles were investigated in recent papers [1,2].

The following sections discuss the properties of polarization, spectral and angular distribution of radiation at axial channeling. The calculations are made by use of classical mechanics and classical electrodynamics in assumption that the relativistic channeled particle excites high levels of vibrational energy of atoms and the distance between the energy levels is much less than the energy of excitation. The relaxation of excitation due to emission of the photons is described as exponential decrease of the amplitude of oscillations. This relatively simple method makes it possible to describe the general properties of radiation of the lattice atoms. Of course, the calculations aimed at experimental investigations of the phenomenon must be carried out by use of quantum mechanical methods for specific structure of a crystal.

2. Spectral and angular distribution of the radiation

The trajectory of a charged particle at the axial channeling is a rather complicated curve which resembles a precessing elliptical orbit [3–5]. In order to have a possibility of analytical study of the properties of radiation we define a spiral-shaped trajectory:

$$x = Vt, \quad y = R\cos\Omega t, \quad z = R\sin\Omega t. \tag{1}$$

We assume that $\Omega R \ll V$ and the particle is ultrarelativistic one, so that $1 - V^2/c^2 \ll 1$. The X-axis coincides with the crystallographic axis.

It is shown in [1] that a relativistic particle with a charge e, passing by the nucleus of an atom, transmits to it the momentum p which is equal to $p = eq/ac$, and it is directed along the line connecting the particle with the nucleus. Here q is the charge of the nucleus, shielded by the inner electrons, a is the shortest distance between the particle trajectory and the nucleus. As a result, the particle excites oscillations of the atom according to the law

$$y_a = A\cos\chi\sin\omega_0\left(t - \frac{x_a}{V}\right), \tag{2}$$

$$z_a = A\sin\chi\sin\omega_0\left(t - \frac{x_a}{V}\right), \tag{3}$$

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http://dx.doi.org/10.1016/j.nimb.2015.04.045
0168-583X/

where $A = eq/mc\omega_0 R$ is the amplitude of oscillations, m is the mass of the atom, ω_0 is the frequency of the exited vibrational mode, the angle $\chi = \Omega x_a/V$ determines the direction of oscillations of the atom with coordinate $x_a = \text{const}$ as shown in Fig. 1. The coordinate x_a is measured from the boundary of the crystal. Eqs. (2) and (3) are valid for $t \geqslant x_a/V$. Before that time $y_a = z_a = 0$. The origin of the time coordinate is chosen at the instance when the channeling particle enters the crystal. We introduce now the attenuation coefficient α in order to take into account the oscillation damping caused by radiation and energy transfer to the neighbouring atoms, and write the law of motion of the atom in the form

$$y_a = A\cos\frac{\Omega x_a}{V} f(x_a, t),$$
$$z_a = A\sin\frac{\Omega x_a}{V} f(x_a, t),$$
$$f(x_a, t) = \sin\omega_0\left(t - \frac{x_a}{V}\right)\exp\left[-\alpha\left(t - \frac{x_a}{V}\right)\right].$$

Let us find the Fourier expansion of the electric field of the atomic radiation. We use the dipole approximation [6] since the atom can be considered as a nonrelativistic one

$$\boldsymbol{E}(\omega, x_a) = \frac{1}{r_a c^2}\int_{x_a/V}^{\infty}\left[\left[\ddot{\boldsymbol{d}}(t')\boldsymbol{n}\right]\boldsymbol{n}\right]e^{i\omega t}\mathrm{d}t, \tag{4}$$

where r_a is distance from the atom to an observer, $\boldsymbol{n}$ is the unit vector in the direction of radiation, $\boldsymbol{d}(t')$ is the dipole moment of the atom at time $t' = t - r_a/c$. We can put in the denominator of this formula $r_a = r$, where r is the distance from the coordinate origin to the observer. But we have to keep the coordinate of atom in t', in order to take properly into account the effects of coherence: $r_a = r - x_a n_x$.

The main part of atomic radiation lies at the frequencies close to the vibrations frequency. The corresponding wavelength is much greater than the distance between the neighbouring atoms. Hence, addition of the fields generated by different atoms of the crystallographic axis can be replaced by integration over x_a:

$$\boldsymbol{E}(\omega) = \frac{1}{b}\int_0^L \boldsymbol{E}(\omega, x_a)\mathrm{d}x_a,$$

where b is the distance between atoms in the chain, L is the channeling length of the particle. We present the result of integration in the spherical coordinate system (r, ϑ, ϕ), with the polar angle ϑ measured from the X-axis, and the azimuthal angle ϕ – from the Y-axis

$$E_\phi(\omega) = I(\omega)(\cos\phi + i\xi\sin\phi), \tag{5}$$
$$E_\vartheta(\omega) = I(\omega)\cos\vartheta(\sin\phi - i\xi\cos\phi), \tag{6}$$
$$I(\omega) = \frac{2qA\omega_0\sin\pi N\xi(\omega_0^2 + \alpha^2 - 2i\alpha\omega)}{rcb\Omega(1-\xi^2)(\omega_0^2 - \omega^2 + \alpha^2 + 2i\alpha\omega)}.$$

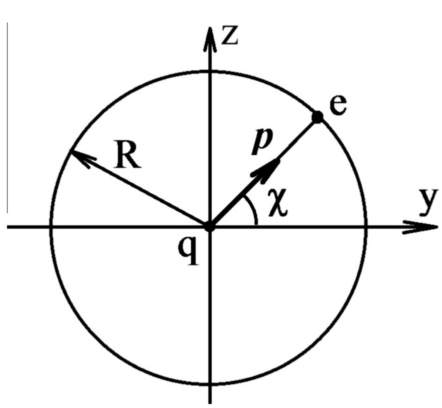

Fig. 1. Projection of the trajectory of a channeled particle on the plane XY. Vector $\boldsymbol{p}$ is the momentum of the atom gained after interaction with the particle.

Here $N = L\Omega/2\pi V$ is the number of oscillations of the channeled particle, which we assume to be an integer; $\xi = \omega/\Omega'$, $\Omega' = \Omega/(1 - \beta\cos\vartheta)$ is the frequency of oscillations of the channeling particle shifted by the Doppler effect; $\beta = V/c$. Taking the square of module of the last expressions and adding up the polarization components, we obtain the energy $\mathrm{d}\mathcal{E}$, emitted into the solid angle $\mathrm{d}o$ in the frequency range $\mathrm{d}\omega$

$$\frac{\mathrm{d}\mathcal{E}(\omega)}{\mathrm{d}o\mathrm{d}\omega} = \frac{q^4e^2\left[\left(\omega_0^2+\alpha^2\right)^2 + 4\omega^2\alpha^2\right]}{\pi^2c^3b^2m^2R^2\Omega^2}\frac{\sin^2\pi N\xi}{\left(1-\xi^2\right)^2} \times\frac{\xi^2 + \cos^2\vartheta + (1-\xi^2)\sin^2\vartheta\cos^2\phi}{\left(\omega_0^2 - \omega^2 + \alpha^2\right)^2 + 4\alpha^2\omega^2}. \tag{7}$$

The coherence of this radiation is expressed by the factor $\sin^2\pi N\xi/(1-\xi^2)^2$ which gives a narrow line in the emission spectrum with the intensity proportional to $N^2/b^2 \sim L^2/b^2$ that is proportional to the square of the number of atoms involved in the radiation process. Dependence on the angle ϕ is due to the fact that the particle makes a finite number of revolutions around the crystallographic axis. Obviously, when $N \to \infty$, the parameter ξ tends to unity and radiation becomes axially symmetric. Near resonance $(\xi = 1)$ radiation is also axially symmetric.

Dependence of the emitted energy on the frequencies Ω' and ω is of typical resonant character: if the coefficient of attenuation of atomic oscillations vanishes $(\alpha \to 0)$ then the energy of radiation at a frequency $\omega = \omega_0$ tends to infinity. Thus, at a reasonably small α the resonance occurs at $\omega = \omega_0$. Having in mind that the main part of radiation is emitted in the vicinity of the frequency $\omega = \Omega'$, the condition of resonant amplification of the radiation at angle ϑ can be written as the ratio between the frequency of exited vibrational mode and the frequency of oscillation of the channeled particle:

$$\Omega = \omega_0(1 - \beta\cos\vartheta). \tag{8}$$

This condition can be satisfied if the frequencies Ω and ω_0 satisfy the inequalities $\omega_0(1-\beta) \leqslant \Omega \leqslant \omega_0(1+\beta)$. Note that the set of oscillators, each being at rest, emits the radiation as if it is a moving relativistic particle – the main part of radiation is directed forward along the average velocity of the channeled particle in a narrow cone of order $\sqrt{1-\beta^2}$.

Fig. 2 shows the angular distribution of the radiation of the atomic chain excited at axial channeling of a particle moving with the velocity $\beta = 0.8$ and frequency $\Omega = \omega_0/4$.

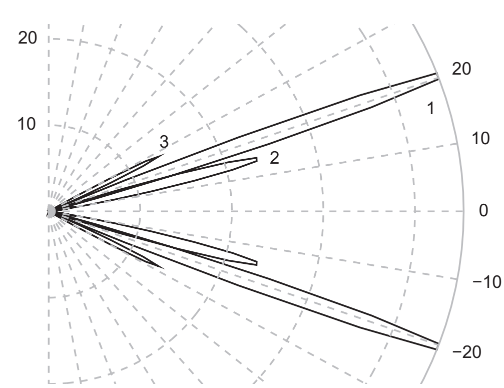

Fig. 2. The radiation pattern for three frequencies lying near the resonant frequency: $\omega = \omega_0$ (1); $\omega = 1.1\omega_0$ (2); $\omega = 0.85\omega_0$ (3). $N = 10$, $\alpha = 0.1\omega_0$.