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Multiple scattering of proton via stochastic differential equations

M.R. Kia, Houshyar Noshad *

Department of Energy Engineering and Physics, Amirkabir University of Technology (Tehran Polytechnic), P.O. Box 15875-4413, Hafez Avenue, Tehran, Iran

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ABSTRACT

Multiple scattering of protons through a target is explained by a set of coupled stochastic differential equations. The motion of protons in matter is calculated by analytical random sampling from Moliere and Landau probability density functions (PDF). To satisfy the Vavilov theory, the moments for energy distribution of a 49.1 MeV proton beam in aluminum target are obtained. The skewness for the PDF of energy demonstrates that the energy distribution of protons in thin thickness becomes a Landau function, whereas, by increasing the thickness of the target it does not follow a Gaussian function completely. Afterwards, the depth-dose distributions are calculated for a 60 MeV proton beam traversing soft tissue and for a 160 MeV proton beam travelling through water. The results prove that when elastic scattering is taken into account, the Bragg-peak position is decreased, while the dose deposited in the Bragg region is increased. The results obtained in this article are benchmarked by comparison of our results with the experimental data reported in the literature.

1. Introduction

Development of accurate algorithms for heavy charged particle transport through matter is important due to its applications in therapy and high RBE compared to conventional radiotherapy with photons and electrons. In general, inelastic collision and elastic scattering are two principle features characterizing the passage of heavy charged particles through matter. Inelastic collision was first calculated by Bohr using the classical argument and later by Bethe-Bloch taking into account quantum mechanics [1,2]. This process for a beam of particles is subject to energy fluctuation, because of the stochastic nature of the energy loss process. Landau demonstrated that the energy probability distribution in thin thickness is highly asymmetric with a pronounced high energy tail [3]. Later Vavilov predicted that the PDF of energy has a shape between the Gaussian and Landau function based on the parameter κ [4]. This theory was experimentally verified by Maccabee and Tschalar [5,6]. In multiple scattering where the energy loss is negligible, Moliere expressed the polar angle distribution as series for investigation on charged particle deviation [7]. Moreover, Bethe presented Moliere distribution function for small angles in a simple mathematical approach [8]. Moliere function can be simplified in Gaussian form if $\Delta x/L_r > 10^{-3}$, where Δx and L_r are the thickness and radiation length of absorber, respectively [9]. In this case, Noshad employed the results of the TRIM computer code and studied beam deviation in matter [10]. Later Mertens et al., derived an analytical method for flux of particles with the Gaussian PDF to analyze the beam deviation [11]. One can find a semi-analytical method based on the Fermi–Eyges equation for calculation of dose distribution [11,12] developed by Hollmark et al. [13]. The results of considering inelastic scattering for dose distribution are investigated in [14–16].

In this article, we derived a set of coupled stochastic differential equations in order to investigate on inelastic collision and elastic scattering by analytical random sampling from the Moliere and Landau PDFs. Besides, the evolution for the PDF of energy as well as the depth-dose distribution are investigated in different materials. We also studied the effect of elastic scattering on depth-dose distribution via the moments of the energy PDF. The results obtained from our stochastic model are in good agreement with the experimental data.

2. Theory and methods

2.1. Inelastic collision

With due attention to the stochastic nature of energy loss for passage of a proton beam through matter, the following formalism has been considered for the stochastic differential equation for evolution of energy $T_i(x)$ in depth x and thickness interval Δx,

$$T_i(x+\Delta x) = T_i(x) + \xi\left(\lambda + \ln\frac{\xi 2mc^2\beta_i(x)^2}{(1-\beta_i(x)^2)I} - \beta_i(x)^2 - C + 1\right). \quad (1)$$

* Corresponding author. Tel.: +98 21 64545254; fax: +98 21 66495519.
E-mail address: hnoshad@aut.ac.ir (H. Noshad).

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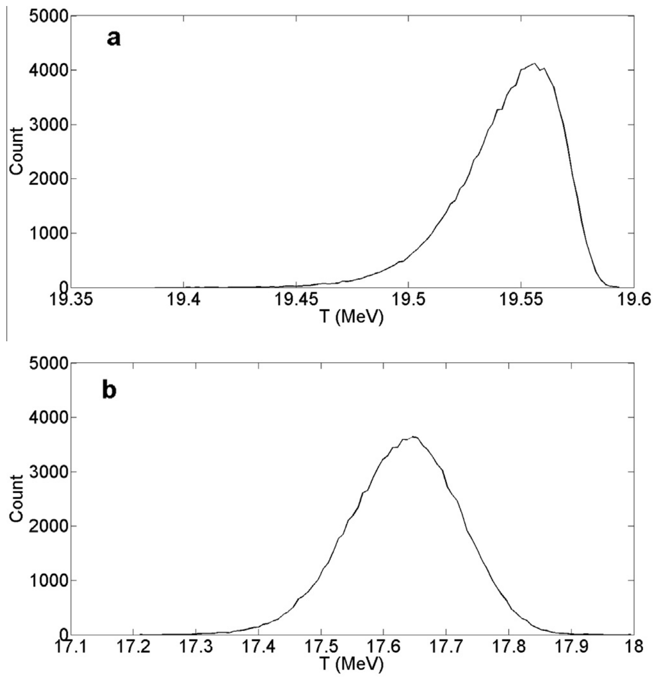

Fig. 1. The energy probability density function for a 19.68 MeV proton beam in an aluminum foil at $x = 0.0051$ g/cm^2 in (a) and $x = 0.099$ g/cm^2 in (b).

In the relation, $C = 0.5772156$ is the Euler–Mascheroni constant and $\beta_i(x)$ is the velocity coefficient of the ith charged particle at step x. The parameter ξ and λ denote the mean energy loss in thickness Δx and Landau parameter, respectively. The mean excitation potential of target material I can be obtained from theoretical formalism and experimental data [19,20]. In order to employ the Landau PDF, the value of thickness Δx is not constant and it is calculated as follows

$$\Delta x = k\frac{1.022\beta^4 A}{(1-\beta^2)0.1535\rho Z}, \tag{2}$$

where k is a constant. Our results are in good agreement with the experimental data with $k = 0.007$ for water and tissue [21]; whereas, $k = 0.00648$ and 0.00640 correspond to aluminum and silicon, respectively [22,23]. To obtain λ, two random numbers $\zeta_1 \in [0, 0.9931]$ and $\zeta_2 \in [0, 1]$ are generated. If $0 \leqslant \zeta_1 < 0.0141$, then λ is obtained as follows

$$\begin{aligned}\lambda = {} & 10^6(0.15948\zeta_2^{15} - 1.277\zeta_2^{14} + 4.672\zeta_2^{13} - 10.35\zeta_2^{12} \\ & + 15.50\zeta_2^{11} - 16.60\zeta_2^{10} + 13.10\zeta_2^{9} - 7.749\zeta_2^{8} + 3.452\zeta_2^{7} \\ & - 1.156\zeta_2^{6}) + 28745\zeta_2^{5} - 5202\zeta_2^{4} + 6621\zeta_2^{3} - 563\zeta_2^{2} \\ & + 30.50\zeta_2 - 3.4.\end{aligned} \tag{3}$$

Moreover, if $0.0141 \leqslant \zeta_1 < 0.1136$, the procedure can be followed as

$$\begin{aligned}\lambda = {} & -3.052\zeta_2^{8} + 14.42\zeta_2^{7} - 29.31\zeta_2^{6} + 33.77\zeta_2^{5} - 24.67\zeta_2^{4} \\ & + 12.33\zeta_2^{3} - 4.725\zeta_2^{2} + 2.226\zeta_2 - 2.\end{aligned} \tag{4}$$

If $0.1136 \leqslant \zeta_1 < 0.6642$, the algorithm can be continued as below

$$\begin{aligned}\lambda = {} & -0.5071\zeta_2^{8} + 4.565\zeta_2^{7} - 11.78\zeta_2^{6} + 16.26\zeta_2^{5} - 13.35\zeta_2^{4} \\ & + 8.298\zeta_2^{3} - 3.103\zeta_2^{2} + 3.622\zeta_2 - 1,\end{aligned} \tag{5}$$

and if $0.6642 \leqslant \zeta_1 < 0.9435$, then the value for λ can be calculated from the following prescription

$$\begin{aligned}\lambda = {} & 293.86\zeta_2^{8} - 968.52\zeta_2^{7} + 1340.7\zeta_2^{6} - 988.77\zeta_2^{5}8 + 419.7\zeta_2^{4} \\ & - 97.852\zeta_2^{3} + 14.704\zeta_2^{2} + 3.1669\zeta_2 + 3.\end{aligned} \tag{6}$$

Finally, if $0.9435 \leqslant \zeta_1 \leqslant 0.9934$, the value for λ can be determined as follows

$$\begin{aligned}\lambda = {} & 16956.88\zeta_2^{10} - 73891.79\zeta_2^{9} + 139407.7\zeta_2^{8} - 148468.2\zeta_2^{7} \\ & + 97854.35\zeta_2^{6} - 41107.26\zeta_2^{5} + 10945.64\zeta_2^{4} - 1745.207\zeta_2^{3} \\ & + 166.5348\zeta_2^{2} + 11.37218\zeta_1 + 20.\end{aligned} \tag{7}$$

According to the above Eqs. (3)–(7), λ is obtained via the aforementioned analytical functions. The associated error for random sampling of parameter λ in the interval $-3.4 \leqslant \lambda \leqslant 150$ is less than 1%

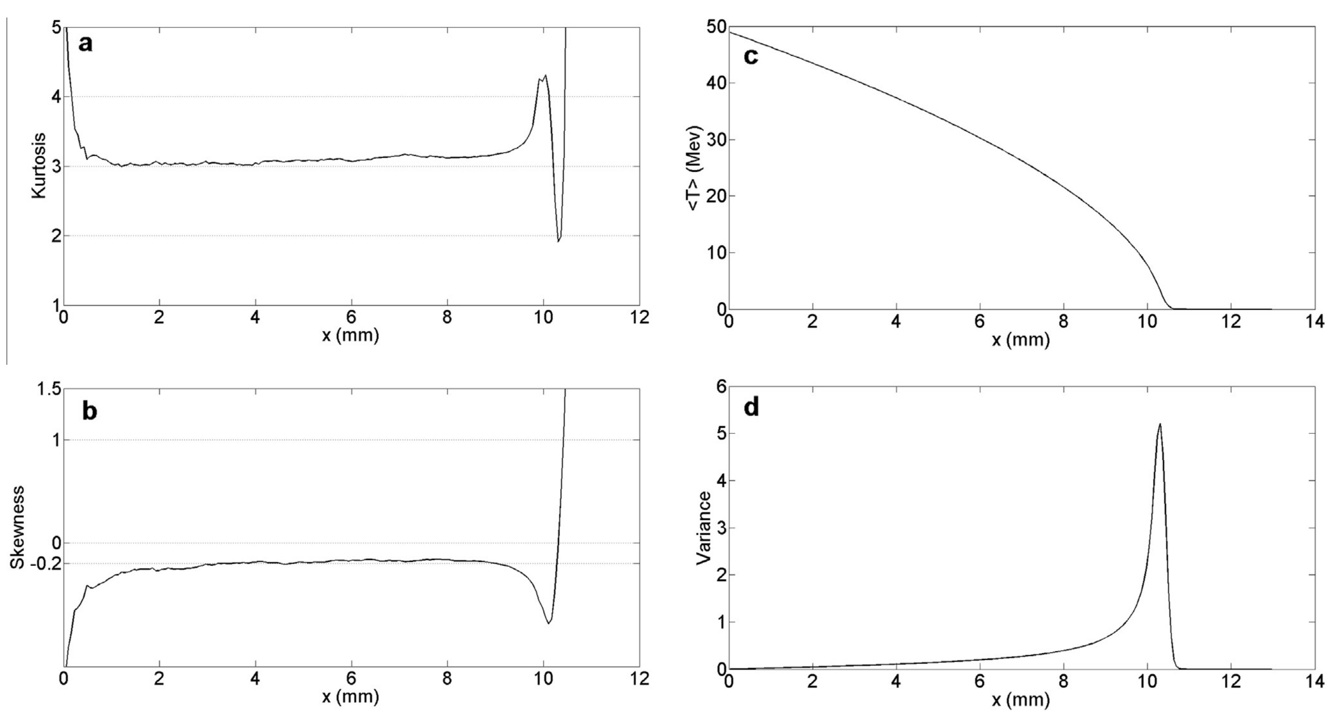

Fig. 2. The moments for PDF of the energy for a 49.1 MeV proton beam in an aluminum target.