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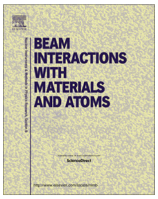

BEAM INTERACTIONS WITH MATERIALS AND ATOMS

Determination of half-value thickness of aluminum foils for different beta sources by using fractional calculus

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ABSTRACT

Reduction of beta-ray intensity with respect to thickness of absorber material exhibits a non-exponential behavior due to the different types of the energy loss processes and many different fractal-like paths followed by beta particles in material. According to Caputo formalism of fractional calculus, the reduction process of beta-ray intensity is governed by using a simple fractional differential equation of order $\alpha \approx 0.31$. The solution is obtained in terms of Mittag–Leffler function which depends on a mass attenuation coefficient μ_m and a fractional order α that can be considered as a measure of fractality of absorbing material. In the experimental part of the study, ^{99}Tc, ^{36}Cl, ^{14}C, ^{210}Pb and ^{147}Pm radioisotopes have been used as beta sources. In the framework of fractional calculus approach, the experimental and calculated half-value thicknesses of all samples have been obtained in agreement with each other.

1. Introduction

Beta particles penetrating in absorbing material interact with the electrons and the nuclei present in material through the electromagnetic force. They undergo elastic collisions with electrons and nuclei due to the electromagnetic fields of these objects. Hence, beta particles can lose their energies by transferring an amount of energy to electrons along their trajectories (energy loss due to ionization) or change their directions due to the collisions with nuclei (multiple scattering). Especially, scattering and back scattering events, that give rise to deviation from simple exponential dependence of energy loss process, have a special importance for low energy electrons. Due to these events, the beta particles penetrate into absorbing material by following different fractal-like (erratically twisted) paths. Since the energy loss processes or the nature of the interactions differ from one beta particle to the other, the distances travelled by each of the beta particles in material are different from each other. In other words, all the beta particles with same kinetic energy follow different fractal-like paths and do not have the same range. Also, for a beta particle, the actual length of the trajectory is usually longer than the mean path of the beta-beam. For this reason, a well-defined range is not obtained for a beta-particle. An average value can be obtained from the relation that the number of beta particles penetrating through an absorbing material decreases with the thickness of material. For this aim, the beta-ray intensity can be considered as a function of distance travelled by beta particles in material [1].

If a beam of beta particles with initial intensity I_0 penetrates in material, the intensity loss of beta-beam with respect to thickness of material is governed by $\frac{dI(x)}{dx} = -I(x)N\sigma$, where $\frac{dI(x)}{dx}$ is the intensity loss of beta-beam per unit length, N is the number of scattering centers per unit volume, σ [cm^2] is a quantity that characterizes the type of interactions of beta particles in material and called as cross section. The term $N\sigma$ is defined as linear attenuation coefficient, $\mu = N\sigma$ [cm^{-1}]. The mean free path of a beta particle before the first collision is defined by $\lambda = 1/\mu$ [cm]. In literature, the intensity loss of beta-beam per unit length is generally divided by the density, thus the reduction process of beta-ray intensity is governed by $\frac{dI(x)}{dx} = -\mu_m I(x)$, where μ_m (cm^2/g) is the mass attenuation coefficient defined by the ratio of linear attenuation coefficient to density of material $\mu_m = \mu/\rho$, and x (g/cm^2) represents the distance travelled by beta-particles in absorber material. In present study, this equation is called as the standard attenuation equation, and the solution of this equation gives the intensity of un-scattered beta-beam in material as a function of distance, $I(x) = I_0 \exp(-\mu_m x)$. Half-value thickness of an absorbing material is the thickness where the beta-ray intensity takes the half-value of initial intensity

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$I_0/2$, and can be obtained as $x_{1/2} = \ln 2/\mu_m$ by using the solution of standard attenuation equation.

Transitions of beta particles through different materials have been the subject of many studies [2–7]. The mass attenuation coefficient and the half-value thickness of absorbing material have a special importance in the theory of radiation physics. In this context, several studies have been employed to explain the physical characteristics of these terms. The research topics of these studies can be summarized as follows: a semi-empirical relation between the mass attenuation coefficient and the maximum energy of electron is given by Ram et al. [8]. Theoretical calculation of monoenergetic electron and positron ranges in *Al*, *Cu*, *Sn*, *Yb* and *Pb* is done by Batra and Sehgal [9]. Another empirical formula and a theoretical discussion of the mass attenuation coefficient are given by Burek and Chocyk [10]. A comprehensive theoretical method for mass attenuation coefficients of *Al*, *Cu* and *Au* is discussed by Gurler and Yalcin [11]. Investigation of the transmission of beta particles through *Al* for different arrangements of point source, absorber and detector is made by Gurler and Yalcin [12]. The beta-ray spectra after penetrating through absorbing materials are observed for different sources by Kawada et al. [13]. The mass attenuation coefficient of brass, cardboard and *Al* is determined experimentally by using $^{90}Sr/^{90}Y$ source by La Rocca and Riggi [14]. A timing method approach is taken to determine the mass attenuation coefficients for different absorbers by Ermis and Celiktas [15]. A semi-empirical model was derived for a ^{90}Sr beta-particle transmission for Al by Gardner et al. [16]. The attenuation coefficients for beta rays were obtained with the solution of linear transport equation for gold absorbers by Idoeta [17]. Moreover, the mass attenuation coefficients were investigated using X-ray and gamma rays by different studies [18–21].

Theoretical predictions about the observed physical processes may not be consistent with the experimental results. This inconsistency can be due to certain interactions and dissipations that are not taken into account in calculations. In literature, in order to achieve more accurate description of the experimental results, some additional terms are usually added to the equations manually. However, this approach is not based on a consistent mathematical formalism. Therefore, the inconsistency between the theoretical and experimental results is mainly due to the mathematical formalism that theory based on. In this context, fractional calculus, which deals with integrals and derivatives of arbitrary order, allows to describe physical processes in a more realistic manner. From this perspective, in our previous study [22], the attenuation equation has been considered with the first order space derivative changed to a Caputo fractional derivative of order $\alpha \approx 0.31$, namely the fractional attenuation equation. In this way, experimental and theoretical half-value thicknesses of aluminum absorbers have been obtained in consistent with each other for ^{137}Cs, ^{204}Tl and $^{90}Sr/^{90}Y$ beta sources. In this study, the half-value thicknesses of aluminum absorbers are studied both experimentally and theoretically by making use of ^{99}Tc, ^{36}Cl, ^{14}C, ^{210}Pb and ^{147}Pm radioisotopes. In the course of the standard calculations, it can be seen that the theoretical half-value thicknesses of aluminum absorbers obtained by using standard attenuation equation are different from the experimental ones. According to our opinion, the reasons of this difference are the fact that each beta particles penetrating in absorbing material follow different fractal-like paths due to the different types of interactions. But, the standard attenuation equation only governs an exponential behavior which corresponds to an average observation, and is not sufficient to describe the process in a more realistic manner. In order to compensate this inconsistency, instead of the well-known first order attenuation equation, the space-fractional attenuation equation of order $\alpha \approx 0.31$, namely ${}_0^C D_x^{0.31} I(x) = -\mu_m^{0.31} I(x)$, has been solved by using Laplace transform technique, where ${}_0^C D_x^{0.31} I(x)$ is the Caputo fractional derivative operator of order 0.31. Here, the fractional order $\alpha \approx 0.31$ is considered as a measure of fractality of the absorber material (Aluminum). As expected, in the course of the fractional approach, the obtained theoretical results are well compatible with experimental values.

The content of the paper is organized as follows: the fractional calculus and the space-fractional form of the attenuation equation are introduced briefly in Section 2. The experimental procedure is reported in Section 3. The calculated results are given and discussed in Section 4. In the last section, the conclusions are summarized.

2. Space-fractional attenuation equation

Intensity loss of beta-beam with respect to the distance travelled by beta particles can be considered as a relaxation process. If the intensity of un-scattered beta-beam in material at x position is represented by $I(x)$, then $\omega(x) = I(x)/I(0)$ is the relaxation function which monotonically decreases with x. Here, $I_0 = I(0)$ is the initial beta-ray intensity, thus $\omega(0) = 1$ can be regarded as an initial condition. In the standard theoretical calculations, the behavior of this relaxation process is regarded as exponential type, and the relaxation function is assumed to satisfy the following standard attenuation equation:

$$\frac{d\omega(x)}{dx} = -\mu_m \omega(x). \tag{1}$$

Hence, the solution of Eq. (1) is obtained as $\omega(x) = \exp(-\mu_m x)$. But the experimental results show that, this type of relaxation process slightly deviates from the exponential behavior in most cases (see Figs. 1 of this study and Ref. [22]). In order to represent this relaxation process of beta-ray intensity in a more accurate manner, a stretched exponential or a power-law behavior can be more convenient than exponential one. Therefore, in accordance with experimental observations, the intensity loss of beta-beam with respect to distance is not described by a first order differential equation.

The solution of the standard attenuation equation corresponds to only a simple idealization of a real process, and represents only an average case of the process over the spectrum of mass attenuation coefficients. For a realistic description, mass attenuation coefficients, that characterize the inner structure of absorber material and the types of interactions, should be taken into consideration in calculations. In an intensity loss process of beta-beam, beta particles can lose energy in many different ways, for example, by ionizing and exciting atoms along their trajectory and by bremsstrahlung effect (for higher energies). Since beta particles can penetrate into a considerable depth in absorber, they also can be affected by multiple scattering [1]. The mass attenuation coefficients, which correspond to these effects, should be placed in an equation that governs the intensity loss process of beta-beam in realistic manner. Furthermore, each beta particles penetrating in absorbing material follow different fractal-like paths due to the structure of absorber material and the types of interactions that affect the beta particles. Therefore, it is plausible that the inner structure of absorbing material can be considered as a non-Euclidean fractal space. In this fractal space, since the energy loss processes are analogously repeated on different paths travelled by beta particles, the intensity loss of a beta-beam is based on self-similar processes. Additionally, the lengths of paths followed by beta particles somewhat differ from one beta particle to the other, thus, the differences of the lengths of the paths can be interpreted as a scaling change.

Scaling change and self-similarity have a special importance in fractals. Beta particles in absorbing material exhibit a fractal