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Interaction of relativistic electrons with an intense laser pulse: High-order harmonic generation based on Thomson scattering

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ABSTRACT

We investigate nonlinear Thomson scattering as a source of high-order harmonic radiation with the potential to enable attosecond light pulse generation. We present a new analytic solution of the electron's relativistic equations of motion in the case of a short laser pulse with a sine-squared envelope. Based on the single electron emission, we compute and analyze the radiated amplitude and phase spectrum for a realistic electron bunch, with special attention to the correct initial values. These results show that the radiation spectrum of an electron bunch in head-on collision with a sufficiently strong laser pulse of sine-squared envelope has a smooth frequency dependence to allow for the synthesis of attosecond light pulses.

1. Introduction

Attosecond physics [1–3] has been an established field of research for more than a decade, providing revolutionary experimental results on truly atomic time scales [4–6]. One of its most important methods is a pump and probe experiment with single attosecond light pulses, which can be routinely achieved with high-order harmonic generation (HHG) on noble gas atoms by suitable few-cycle laser pulses. However, gas HHG has well-known limitations and thus new methods capable to produce bright attosecond (or shorter) light pulses are of great importance to this research field.

The interaction of a relativistic electron beam with an intense laser pulse, i.e. nonlinear Thomson scattering [7–9], is a promising method for the generation of high-order harmonics and potentially attosecond light pulses. In the context of nonlinear Thomson scattering [10,11], the emitted power spectrum is in the focus of most of the publications, for which there are well-known analytical [11] and numerical [12] calculations. Chen et. al. also pointed out that, in case of a few-cycle laser pulse, the emission spectrum should be calculated with the formula involving the acceleration of the electron [13,14].

The equations of motion of a single relativistic electron in an electromagnetic plane wave in a vacuum have well-known exact solutions.. However, the generalization of this solution for several electrons interacting with a short laser pulse needs special attention, in particular, regarding the correct initial conditions. The importance of space-like initial data was first highlighted in an earlier work of one of the authors [15]. In the present paper, we address this problem in the framework of classical electrodynamics, based on [15], with a modified configuration. We demonstrate that the collectively emitted amplitude and phase spectrum allows to create a single attosecond pulse with the help of proper filtering.

In Section 2, we recall the classical equations of motion for a single electron in head-on collision with a strong ultrashort laser pulse and give their analytical solution assuming laser pulses with a sudden switch-on or a sine-squared envelope. Then we determine the initial values of these solutions from the usual (spatially separated) initial conditions explicitly. In Section 3, we calculate and present the amplitude and phase spectra of the emitted radiation and an attosecond pulse shape corresponding to a filtered spectral region. Finally, we summarize our results and conclusions in Section 4.

2. Equation of motion, electron trajectories

We consider a classical relativistic electron in head-on collision with a strong ultrashort laser pulse, modeled as an electromagnetic plane wave traveling in the z-direction, with linear polarization along the x-direction:

$$\mathbf{E}(\Theta) = E_0 \hat{\mathbf{e}}_x f(\Theta) \sin(\omega_L \Theta + \varphi_0), \quad (1)$$

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where

$$\Theta = t - \frac{z}{c}, \tag{2}$$

denotes the argument of the plane wave, E_0 is the amplitude of the electric field strength and ω_L is the central frequency of the plane wave. The $f(\Theta)$ represents the pulse envelope function. Using $\mathbf{B} = \mathbf{n}_L \times \mathbf{E}$, where $\mathbf{n}_L$ is the unit vector pointing in the positive z-direction, the Newton–Lorentz equations for the electron of charge e and mass m are the following:

$$m\frac{d\mathbf{u}}{d\tau} = \frac{e}{c}\left[u^0\mathbf{E}(\Theta) + \mathbf{n}_L(\mathbf{u}\cdot\mathbf{E}(\Theta)) - \mathbf{E}(\Theta)(\mathbf{n}_L\cdot\mathbf{u})\right], \tag{3}$$

$$m\frac{du^0}{d\tau} = \frac{e}{c}\mathbf{E}\cdot\mathbf{u}, \tag{4}$$

where $(u^0, \mathbf{u}) = (\gamma c, \gamma\mathbf{v})$ is the four-velocity of the electron, $\gamma \equiv \left(1 - |\mathbf{v}|^2/c^2\right)^{-1/2}$ is the Lorentz-factor and $d\tau = dt/\gamma$ is the proper time of the electron. Comparing the z-component of (3) with (4), it follows that $u^0 - u^3$ is a constant of motion:

$$u^0 - u^3 = \frac{d}{d\tau}(ct - z) = c\frac{d\Theta}{d\tau} = \alpha c, \tag{5}$$

where $\alpha = \gamma(1 - v_z/c)$ denotes a dimensionless constant which is determined by the initial conditions. The connection (5) enables the replacement of the derivatives with respect to the proper time τ by derivatives with respect to the wave argument Θ in (3) and (4), which yields the following equations

$$m\alpha\frac{d^2x}{d\Theta^2} = eE(\Theta), \tag{6}$$

$$\frac{d^2y}{d\Theta^2} = 0, \tag{7}$$

$$\frac{d^2z}{d\Theta^2} = \frac{1}{2c}\frac{d}{d\Theta}\left(\frac{dx}{d\Theta}\right)^2, \tag{8}$$

$$\frac{d^2(ct)}{d\Theta^2} = \frac{1}{2c}\frac{d}{d\Theta}\left(\frac{dx}{d\Theta}\right)^2. \tag{9}$$

2.1. Solution for a laser pulse with sudden switch-on

The equations of motion (6)–(9) have the following analytic solution in the case of a laser pulse which is switched on suddenly at $\Theta = \Theta_0$:

$$x(\Theta) = x(\Theta_0) + \widetilde{W}_{x_0}(\Theta - \Theta_0) - \frac{cv}{\omega}(\sin(\Theta\omega_L) - \sin(\Theta_0\omega_L)), \tag{10}$$

$$y(\Theta) = y(\Theta_0) + W_{y_0}(\Theta - \Theta_0), \tag{11}$$

$$\begin{aligned} z(\Theta) = {} & z(\Theta_0) + \widetilde{W}_{z_0}(\Theta - \Theta_0) + \widetilde{W}_{x_0} v\cos(\Theta_0\omega_L)(\Theta - \Theta_0) \\ & - \widetilde{W}_{x_0}\frac{v}{\omega_L}(\sin(\Theta\omega_L) - \sin(\Theta_0\omega_L)) \\ & + \frac{cv^2}{8\omega_L}(\sin(2\Theta\omega_L) - \sin(2\Theta_0\omega_L)), \end{aligned} \tag{12}$$

$$\begin{aligned} ct(\Theta) = {} & ct(\Theta_0) + \widetilde{W}_{t_0}(\Theta - \Theta_0) + \widetilde{W}_{x_0} v\cos(\Theta_0\omega_L)(\Theta - \Theta_0) \\ & - \widetilde{W}_{x_0}\frac{v}{\omega_L}(\sin(\Theta\omega_L) - \sin(\Theta_0\omega_L)) \\ & + \frac{cv^2}{8\omega_L}(\sin(2\Theta\omega_L) - \sin(2\Theta_0\omega_L)), \end{aligned} \tag{13}$$

where the following quantities are introduced

$$\widetilde{W}_{x_0} = W_{x_0} + cv\cos(\Theta_0\omega_L), \tag{14}$$

$$\widetilde{W}_{z_0} = W_{z_0} - \frac{cv^2}{4}\cos(2\Theta_0\omega_L), \tag{15}$$

$$\widetilde{W}_{t_0} = W_{t_0} - \frac{cv^2}{4}\cos(2\Theta_0\omega_L), \tag{16}$$

with

$$\{W_t(\Theta_0), \mathbf{W}(\Theta_0)\} = \{W_{t_0}, W_{x_0}, W_{y_0}, W_{z_0}\} = \left.\left\{\frac{d(ct)}{d\Theta}, \frac{dx}{d\Theta}, \frac{dy}{d\Theta}, \frac{dz}{d\Theta}\right\}\right|_{\Theta=\Theta_0} \tag{17}$$

being determined by the true initial values, to be discussed in Section 2.3. In these formulae, the following effective intensity parameter is used:

$$v = \frac{\mu}{\alpha}, \tag{18}$$

which is defined in terms of the general intensity parameter

$$\mu = \frac{|e|E_0}{mc\omega}. \tag{19}$$

For a monochromatic plane wave, the value of μ can be easily calculated with the following formula:

$$\mu = 8.5\cdot 10^{-10}\lambda[\mu\mathrm{m}]\sqrt{I_0\left[\frac{W}{\mathrm{cm}^2}\right]}. \tag{20}$$

2.2. Solution for a laser pulse with sine-squared envelope

Now we assume a few cycle laser pulse with a sine-squared envelope function as:

$$f(\Theta) = \begin{cases} \sin^2(\Omega\Theta), & 0 < \Theta < \pi/\Omega, \\ 0, & \text{otherwise}, \end{cases} \tag{21}$$

then φ_0 in (1) is in fact related to the usual carrier envelope phase difference with a shift of $\pi/2$. The equations of motion (6)–(9) have new analytic solutions for these kind of laser pulses, regardless of the value of Ω and φ_0. Here we present such a solution in the case of a 3-cycle pulse (i.e. $\Omega = \omega/6$) with $\varphi_0 = \pi/2$ (called usually cosine pulse in the context of few-cycle pulses):

$$\begin{aligned} x(\Theta) = {} & x(\Theta_0) + \overline{W}_{x_0}\cdot(\Theta - \Theta_0) \\ & + \frac{cv}{16\omega_L}\sum_{n=2}^{4}\frac{3a_n}{n}\left[\cos\left(\frac{n\Theta\omega_L}{3}\right) - \cos\left(\frac{n\Theta_0\omega_L}{3}\right)\right], \end{aligned} \tag{22}$$

$$y(\Theta) = y(\Theta_0) + W_{y_0}(\Theta - \Theta_0), \tag{23}$$

$$\begin{aligned} z(\Theta) = {} & z(\Theta_0) + \overline{W}_{z_0}\cdot(\Theta - \Theta_0) \\ & + \frac{v}{16}\overline{W}_{x_0}\cdot(\Theta - \Theta_0)\cdot\sum_{n=2}^{4}a_n\sin\left(\frac{n\Theta_0\omega_L}{3}\right) \\ & + \frac{v\cdot\overline{W}_{x_0}}{16\omega}\sum_{n=2}^{4}\frac{3a_n}{n}\left[\cos\left(\frac{n\Theta\omega_L}{3}\right) - \cos\left(\frac{n\Theta_0\omega_L}{3}\right)\right] \\ & - \frac{cv^2}{32\omega_L}\sum_{n=1}^{8}\frac{3b_n}{n}\left[\sin\left(\frac{n\Theta\omega_L}{3}\right) - \sin\left(\frac{n\Theta_0\omega_L}{3}\right)\right], \end{aligned} \tag{24}$$

$$\begin{aligned} ct(\Theta) = {} & ct(\Theta_0) + \overline{W}_{t_0}\cdot(\Theta - \Theta_0) \\ & + \frac{v}{16}\overline{W}_{x_0}\cdot(\Theta - \Theta_0)\cdot\sum_{n=2}^{4}a_n\sin\left(\frac{n\Theta_0\omega_L}{3}\right) \\ & + \frac{v\cdot\overline{W}_{x_0}}{16\omega}\sum_{n=2}^{4}\frac{3a_n}{n}\left[\cos\left(\frac{n\Theta\omega_L}{3}\right) - \cos\left(\frac{n\Theta_0\omega_L}{3}\right)\right] \\ & - \frac{cv^2}{32\omega_L}\sum_{n=1}^{8}\frac{3b_n}{n}\left[\sin\left(\frac{n\Theta\omega_L}{3}\right) - \sin\left(\frac{n\Theta_0\omega_L}{3}\right)\right], \end{aligned} \tag{25}$$

where the following quantities are introduced:

$$\overline{W}_{x_0} = W_{x_0} + \frac{cv}{16}\sum_{n=2}^{4}a_n\sin\left(\frac{n\Theta_0\omega_L}{3}\right), \tag{26}$$