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Infrared multiphoton resummation in quantum electrodynamics

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ABSTRACT

Infrared singularities in massless gauge theories are known since the foundation of quantum field theories. The root of this problem can be tracked back to the very definition of these long-range interacting theories such as QED. It can be shown that singularities are caused by the massless degrees of freedom (i.e. the photons in the case of QED). In the Bloch–Nordsieck model the absence of the infrared catastrophe can be shown exactly by the complete summation of the radiative corrections. In this paper we will give the idea of the derivation of the Bloch–Nordsieck propagators, that describes the infrared structure of the electron propagation, at zero and finite temperatures. Some ideas of the possible applications are also mentioned.

1. Introduction

The infrared (IR) limit of quantum electrodynamics (QED) is known to be plagued by singularities caused by the photons. This phenomenon is known as the infrared catastrophe, and it can be found in any quantum field theory (QFT) which involves massless fields. The development of QFTs started around 1930 with QED, therefore, in most of the cases the subjects of the computations were electromagnetic quantities. The methods used for the calculations were mostly the direct extension of the PT from quantum mechanics. Physicists back then, who were doing computations in QED, immediately faced IR divergences when calculating first order perturbative corrections to the Bremsstrahlung process, due to the low frequency photon contributions. The core of the problem lays in the fundamental definition of QED, namely, that we assume the existence of a free theory, i.e. the existence of asymptotic states. However, such states are difficult to define in a theory where we have long-range interactions. As a consequence, one cannot truly define the asymptotic states described by the Fock representation of free theory Hilbert space, on which the PT is performed. Thus, we need to search for a non-perturbative solution to prevent these difficulties. An alternative approach to this problem was provided by Bloch and Nordsieck in 1937 in their remarkable work on treating the infrared problem [1]. The divergencies are caused by the fact that in a scattering process an infinite amount of long wavelength photons are emitted, and these low energy excitations of the photon field are always present around the electron in the form of a "photon cloud". This shows us essentially that the observed particle is in fact very different from the one we call the bare particle: they can be considered as dressed "quasi particle" objects whose interactions cannot be described through PT entirely. In this paper, we will show the emergence of the infrared catastrophe and then we will introduce the Bloch–Nordsieck (BN) model, which was designed in order to imitate the low energy regime of QED. We will discuss the breakdown of the PT due to the IR catastrophe, however, it is possible to obtain the exact full solution by using the Ward–Takahashi identities embedded into the Dyson–Schwinger (DS) equation.

2. The infrared catastrophe

The easiest way to demonstrate the IR catastrophe is the following. Suppose that $E(\omega)$ is a finite amount of electromagnetic energy emitted by an accelerated charge in the frequency band $[\omega, \omega + d\omega]$. Each photon carries an energy of $\hbar\omega$, hence the average number of emitted photons in this band is $\bar{n} = E(\omega)/\hbar\omega$. If we take the limit $\omega \to 0$ the average number of photons will diverge provided that $\lim_{\omega\to 0} E(\omega) \neq 0$ (which is fulfilled). Thus we can see that an infinite number of soft photons are present at any scattering process (c.f. [2–4]). In the following we will apply the convention used by the particle physics community, i.e. $\hbar = c = 1$ in the further computations.

We should get the same result using quantum computations, however, relying on PT gives different result: already in the first order of the PT the probability of emitting one soft photon in a scattering process will diverge logarithmically [2,3]. This is in complete contradiction that we have found with the semi-classical line of thought a few lines above. In fact, it turns out it is not enough to

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take into account the tree level diagrams but we will also need to include the virtual corrections to cancel out these infrared divergencies. This can be done in all orders of PT and by summing up these corrections (combining real and virtual corrections) to infinite order we can obtain a well defined probability measure. More precisely, it can be shown (c.f. [4]) that the probability of emitting n soft photons in the process has the form

$$P_{\Delta E,\omega_{min}}(n) = |\langle p'|S|p\rangle|^2 \frac{1}{n!}\left[\frac{2\alpha}{3\pi}\frac{-q^2}{m^2}\ln\frac{\Delta E}{\omega_{min}}\right]^n e^{-\frac{2\alpha}{3\pi}\frac{q^2}{m^2}\ln\frac{\Delta E}{\omega_{min}}}. \quad (1)$$

Here the first factor is the absolute square of the scattering amplitude without radiation emitted, α is the fine structure constant ($\alpha = e^2/4\pi$, with e being the electric charge); m^2 and q^2 is the electron mass and the transferred four momentum $q = p - p'$ (and $-q^2 > 0$), respectively. There are two energy scales that have been introduced: ω_{min} and ΔE. The former is an artificial IR regulator ("mass" for the photon field) and the second is the resolution of the detector that performs the measuring in the process: photons having energy lower than ΔE are not being detected at all. Hence, we only consider the interval where the photons energy are $\omega \in [\omega_{min}, \Delta E]$. However, (1) will give 0 for any finite n while taking the artificial mass of the photon to zero as we should:

$$\lim_{\omega_{min}\to 0} P_{\Delta E,\omega_{min}}(n) = 0. \quad (2)$$

This means that the probability of emitting any finite number of soft photons during the scattering process is zero. On the contrary: if we perform a summation over all possible photon numbers that can be emitted we will get a finite result[1]

$$P(\Delta E) = \sum_{n=0}^{\infty} P_{\Delta E}(n) = |\langle p'|S|p\rangle|^2 e^{-\frac{2\alpha}{\pi}\frac{-q^2}{m^2}\ln\frac{m}{\Delta E}}. \quad (4)$$

As we can see, indeed, we obtained a finite probability for the process of infinite emitted photons from the quantum computation. However, we still need to keep the sensitivity of the detector finite in order to get this result. Although theoretically it is possible to take the limit $\Delta E \to 0$, however, in reality it will never happen since there is no such as a detector with perfect resolution.

3. The soft photon contribution to the electron structure: the Bloch–Nordsieck model

The BN model was made to give an insight in the analytic structure of the cancellation of the infinities in the IR regime. Investigating this model will lead us to the exact result of the propagator of the fermion which is surrounded by a cloud of photons. Although the result is known long ago [1], and it can be considered as a textbook material [5], we will sketch the derivation used in [6] which can be extended to finite temperatures the most easily [7]. The main idea of the BN model, that simplifies the theory tremendously, is to replace the gamma matrices γ_μ by a four-vector u_μ that can be considered as the four-velocity of the fermion, and hence the fermion field is represented by a scalar field. This simplification is well justified in the IR regime: the soft photons which take part in the interaction will not have enough energy for pair production, moreover, not even enough to flip the spin of the electron. It implies that the photon propagator will not have any corrections, i.e. the exact photon propagator is the free one in this model. Thus the Lagrangian reads

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(iu_\mu D^\mu - m)\Psi, \qquad iD_\mu = i\partial_\mu - eA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (5)$$

where Ψ and A_μ is the fermion and photon field, respectively. The fermionic part of the Lagrangian is Lorentz-covariant, therefore we can relate the results with different u^μ choice by Lorentz transformation. This makes possible to work with $u = (u_0, 0, 0, 0)$ without loss of generality. In fact, we can perform a Lorentz-transformation where $\Lambda u = (u_0, 0, 0, 0)$. Since u^μ is a four-velocity then $u_0 = 1$; if it is of the form $u = (1, \mathbf{v})$, then it is $u_0 = \sqrt{1-\mathbf{v}^2}$. After rescaling the field as $\Psi \to \Psi/\sqrt{u_0}$ and the mass as $m \to u_0 m$, the Lagrangian reads

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(iD_0 - m)\Psi. \quad (6)$$

Using this reference frame simplifies the calculations [6]. If necessary, the complete u dependence can be recovered easily, however, at finite temperatures we need to use numerics if we want to switch to another reference frame due to the lack of the Lorentz symmetry.

3.1. The Bloch–Nordsieck model at $T = 0$

In order to derive the dressed fermionic propagator, we are going to use a system of self-consistent equations which involves the DS equation, Dyson's series and the Ward–Takahashi identities. We will begin with the DS equation; it reads in real and momentum space, respectively:

$$\begin{aligned}\Sigma(x-y) &= -ie^2\int d^4w\int d^4z\,\mathcal{G}(x-w)u^\mu G_{\mu\nu}(x-z)\Gamma^\nu(z;w,y),\\ \Sigma(p) &= -ie^2\int\frac{d^4k}{(2\pi)^4}\mathcal{G}(p-k)u^\mu G_{\mu\nu}(k)\Gamma^\nu(k;p-k,p),\end{aligned} \quad (7)$$

where $G_{\mu\nu}$ is the photon propagator, $\mathcal{G}$ is the fermion propagator and Γ^μ is the vertex function. The diagrammatic representation of Eq. (7) can be seen in Fig. 1. The DS equation describes the self-energy of the fermion. To obtain the full expression, we need to treat $\mathcal{G}$ as the exact fermion propagator and keep the photon propagator undressed (i.e. on tree-level), which coincides with the exact one in the framework of the BN model. The vertex correction is composed from both propagators but it can be simplified using the Ward–Takahashi relations, as we will see.

Now that we have the formula for the self-energy, we need another equation which expresses the fermion propagator as a function of the self-energy. For this purpose Dyson's series can be used [2]. It can be shown that summing up all the radiative

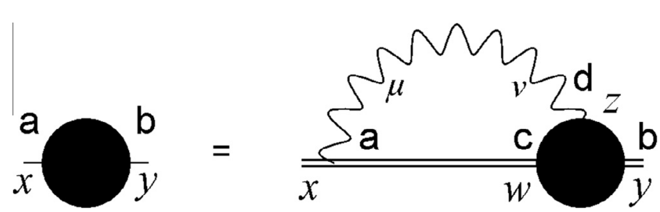

Fig. 1. The diagrammatic representation of the Dyson–Schwinger equation. The double line is for the dressed fermion, the wavy line is for the photon propagator. The black blob denotes the full vertex function. The bold letters are for space–time points and the Greek letters denote the Lorentz indices. At finite temperatures we also need to consider the regular letters which are the Keldysh indices.

[1] This result can be found in [4] where functional techniques are used, however, in [2] the following formula is given for the differential cross section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_0 e^{-\frac{\alpha}{\pi}\ln\frac{-q^2}{m^2}\ln\frac{-q^2}{\Delta E^2}}. \quad (3)$$

Here the first factor corresponds to the hard scattering and in the exponent we can find the famous *Sudakov double logarithm*. The difference between the results in [2,4] originates from the different approximations that are used.