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Fluid Dynamics and Transport Phenomena

Lattice Boltzmann simulation of double diffusive natural convection in a square cavity with a hot square obstacle

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ABSTRACT

Double diffusion convection in a cavity with a hot square obstacle inside is simulated using the lattice Boltzmann method. The results are presented for the Rayleigh numbers 10^4,10^5 and 10^6, the Lewis numbers 0.1, 2 and 10 and aspect ratio A (obstacle height/cavity height) of 0.2, 0.4 and 0.6 for a range of buoyancy number $N = 0$ to -4 with the effect of opposing flow. The results indicate that for $|N| < 1$, the Nusselt and Sherwood numbers decrease as buoyancy ratio increases, while for $|N| > 1$, they increase with $|N|$. As the Lewis number increases, higher buoyancy ratio is required to overcome the thermal effects and the minimum value of the Nusselt and Sherwood numbers occur at higher buoyancy ratios. The increase in the Rayleigh or Lewis number results in the formation of the multi-cell flow in the enclosure and the vortices will vanish as $|N|$ increases.
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1. Introduction

Natural convection occurs because of the buoyancy effect due to temperature gradient. There are also gradients of other scalar quantities such as species concentration in a flow. The flow driven by the joint effect of temperature and species concentration gradients is called double diffusive natural convection (DDNC), which appears in many fields such as astrophysics, oceanography, geology, biology, chemistry and limnology [1], and has many engineering applications such as in crystal growth, energy storage, chemical processes [2].

Natural convection in enclosures has been studied comprehensively [3–5]. Moreover, double diffusive convection has been the subject of study for many researchers [6,7]. DDNC of a non-Newtonian fluid in a shallow horizontal cavity was studied analytically and numerically where the short walls were submitted to uniform heat and salt fluxes and horizontal walls were insulated and impermeable [2]. Double diffusion convection coupled with radiation was numerically studied in a square cavity [8]. In their study the finite volume method was utilized by implementation of a SIMPLER algorithm for coupling of velocity and pressure, and to model radiation heat transfer equation, the discrete ordinate method was used. Nikbakhti and Rahimi [9] numerically studied DDNC in a rectangular cavity. In their study, a part of vertical walls with their length half their cavity height was considered at a constant temperature and concentration. The active part of the left wall had a greater temperature and concentration than the active part of right wall while horizontal walls, and inactive parts of vertical walls had no diffusion. Since placement order of active zones plays a huge role in heat and mass transfer, they considered nine different positions for active parts. For the nine positions the active zones were at the top, middle and bottom.

Recently the Lattice Boltzmann method has successfully substituted the conventional methods such as finite volume method and finite element method. The privilege of this method is its capability in calculating complex geometries, complex boundaries and multiphase flows. Many researchers have studied fluid flow and heat transfer in enclosures and micro-channels using the Lattice Boltzmann method [10–17]. Among them, some authors studied double diffusive natural convection using LBM. Ma [18] proposed a temperature-concentration lattice BGK model to simulate DDNC in a rectangular cavity in the presence of a magnetic field and heat source. In the rectangular cavity, horizontal walls were insulated while vertical walls were set to constant temperature and concentration. A uniform magnetic field was applied in the x direction. DDNC in an open cavity was studied using lattice Boltzmann method [19]. A simple D2Q9 model was used for flow while for temperature and concentration a D2Q4 model was applied. The square cavity has insulated and impermeable horizontal walls while the vertical walls have constant temperature and concentration. In this study only the opposing buoyancy forces were investigated.

While LBM simulation of DDNC was previously investigated by many researchers, to the best knowledge of the authors DDNC in the presence of an obstacle is not yet addressed in the literature. The species concentration induced buoyancy force either aids or opposes the thermally driven flow, determined by the value of the buoyancy ratio, *i.e.* the ratio of the concentration buoyancy force to the thermal buoyancy force. In this paper we investigate double diffusive natural convection in a square cavity in the presence of a hot square obstacle. The effect of aiding flow ($N > 0$) is not the subject of interest.

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2. Method of Solution

A simple D2Q9 scheme is applied for flow, temperature and concentration. Fig. 1 shows geometry of the problem and the boundary conditions. North and south walls are adiabatic (no temperature or concentration diffusion), while left and right boundaries have constant temperature and concentration. All walls of the obstacle have unit temperature and concentration. The Prandtl number is 0.71.

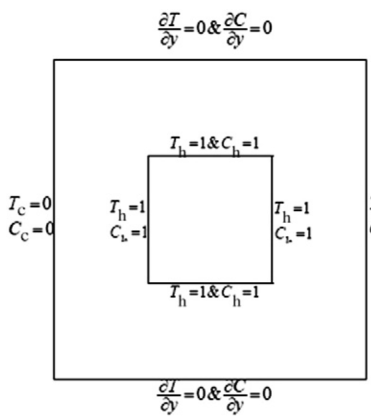

Fig. 1. Cavity geometry and boundary conditions.

Streaming and collision terms of flow are presented as [20]:

$$f_a(x+\boldsymbol{e}_a\Delta t, t+\Delta t) = f_a(x,t) - \frac{\left[f_a(x,t) - f_a^{eq}(x,t)\right]}{\tau} + \boldsymbol{F}_a\Delta t \tag{1}$$

where $f_a(x+\boldsymbol{e}_a\Delta t, t+\Delta t)$ is the streaming part and the right hand side of the equation is the collision term. f_a^{eq} is the equilibrium distribution function and τ is the relaxation time. The equilibrium distribution function f_a^{eq} is given by

$$f_a^{eq}(x) = \omega_a\rho(x)\left[1 + 3\frac{\boldsymbol{e}_a\boldsymbol{u}}{c^2} + \frac{9}{2}\frac{(\boldsymbol{e}_a\boldsymbol{u})^2}{c^4} - \frac{3}{2}\frac{\boldsymbol{u}\ \boldsymbol{u}}{c^2}\right] \tag{2}$$

where ρ and $\boldsymbol{u}$ are the density and microscopic velocity respectively and ω_a is the weight factor which are defined for the D2Q9 model as

$$\omega_a = \begin{cases} {}^4/_9 & a=0 \\ {}^1/_9 & a=1,2,3,4. \\ {}^1/_{36} & a=5,6,7,8 \end{cases} \tag{3}$$

The velocities $\boldsymbol{e}_a$ are [19]

$$\boldsymbol{e}_a = \begin{cases} 0 & & a=0 \\ c(\cos\theta_a, \sin\theta_a) & \theta_a = (a-1)\frac{\pi}{2} & a=1,2,3,4 \\ c\sqrt{2}\left(\cos\theta_a, \sin\theta_a\right) & \theta_a = (a-5)\frac{\pi}{2}+\frac{\pi}{4} & a=5,6,7,8 \end{cases} \tag{4}$$

where $c=\Delta x/\Delta t$, Δx is the lattice space and Δt is the lattice time step size which is set to one.

In Eq. (1), $\boldsymbol{F}_a$ is the force term in each direction and can be defined as [20]:

$$\boldsymbol{F}_a = \omega_a\boldsymbol{F}.\frac{\boldsymbol{e}_a}{c_s^2} \tag{5}$$

where $\boldsymbol{F}$ is

$$\boldsymbol{F} = \rho\mathbf{g}_r(\beta_T\Delta T + \beta_C\Delta C) = \rho\mathbf{g}_r\beta_T\Delta T(1+N). \tag{6}$$

In the above, $\mathbf{g}_r$, β_T and β_C are gravity acceleration, thermal expansion coefficient and concentration expansion coefficient and ΔT and ΔC are temperature and concentration differences respectively. The buoyancy ratio (N) is defined as

$$N = \frac{\beta_C\Delta C}{\beta_T\Delta T}.$$

The macroscopic velocity $\boldsymbol{u}$ and density ρ can be obtained through the first and zeroth moment of the particle distribution f, *i.e.* [20–22]:

$$\boldsymbol{u} = \frac{1}{\rho}\sum\nolimits_{a=0}^{8} f_a\boldsymbol{e}_a \tag{7}$$

$$\rho = \sum\nolimits_{a=0}^{8} f_a. \tag{8}$$

The kinematic viscosity in the D2Q9 method is defined as [20–22]:

$$\upsilon = \left[\tau - \frac{1}{2}\right]c_s^2\Delta t \tag{9}$$

where c_s is the speed of sound defined by $c_s = c/\sqrt{3}$. It should be noted that the above variables ($\boldsymbol{u}, \rho, \ldots$) are lattice quantities which can be related to physical quantities with simple conversion ratios.

Temperature (or concentration) streaming and collision are presented in this manner [19]:

$$g_a(x+\Delta x, t+\Delta t) = g_a(x,t)(1-\omega_s) + \omega_s g_a^{eq}(x,t) \tag{10}$$

where $g_a^{eq}(x,t)$ is the thermal (or concentration) equilibrium distribution function and ω_s is the relaxation time. The thermal (or concentration) equilibrium distribution function is defined as

$$g_a^{eq} = \omega_a\Phi(x,t)\left[1 + \frac{\boldsymbol{e}_a\boldsymbol{u}}{c_s^2}\right] \tag{11}$$

where $\Phi(x, t)$ is either the temperature or the concentration. For D2Q9 model ω_s is given by

$$\omega_s = \frac{1}{3\Gamma + 0.5} \tag{12}$$

where Γ is the diffusion coefficient for temperature (α) or concentration (D). The temperature and concentration can then be calculated at any point in the domain:

$$\Phi(x,t) = \sum_{a=0}^{8} g_a. \tag{13}$$

The average Nusselt number is defined as

$$Nu = \frac{1}{M}\sum_{k=1}^{M} -\frac{\partial T}{\partial X} \tag{14}$$

where ΔX is the dimensionless lattice spacing. The average Sherwood number can be calculated in a similar manner:

$$Sh = \frac{1}{M}\sum_{k=1}^{M} -\frac{\partial C}{\partial X} \tag{15}$$

where M is the number of lattice nodes in Y direction. After the streaming process the distribution functions in the domain are obtained. The distribution functions toward the domain, which are unknown, are then determined by applying the boundary conditions.

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