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Dynamic angle selection in X-ray computed tomography

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ABSTRACT

In X-ray tomography, a number of radiographs (projections) are recorded from which a tomogram is then reconstructed. Conventionally, these projections are acquired equiangularly, resulting in an unbiased sampling of the Radon space. However, especially in case when only a limited number of projections can be acquired, the selection of the angles has a large impact on the quality of the reconstructed image. In this paper, a dynamic algorithm is proposed, in which new projection angles are selected by maximizing the information gain about the object, given the set of possible new angles. Experiments show that this approach can select projection angles for which the accuracy of the reconstructed image is significantly higher compared to the standard angle selections schemes.

1. Introduction

Tomography has applications ranging from 3D imaging of nano-materials by electron microscopy to the reconstruction of accretion disks from astronomical observations. In many of these applications, it is highly desirable to reduce the number of projections taken, or it is even impossible to acquire many projections. In image-guided radiotherapy, for example, a patient is being imaged for several times posing a serious radiation safety concern [1]. In astro-tomography, only a few satellites are capable of imaging the corona of the sun, leading to long acquisition times. In electron tomography, the electron beam gradually damages the object, also imposing a restriction on the number of projections that can be acquired [2].

When an image is being reconstructed from a small number of projections, the angles from which these projections will be acquired will significantly influence the reconstruction quality. In [3], it was shown that the quality of the reconstructions can be highly dependent on the projection angles in binary tomography. In that paper, an algorithm was proposed for identifying optimal projection angles based on a blueprint image known to be similar to the scanned object, which can be readily applied in the field of non-destructive testing. For the more general case of grey scale tomography, a framework was proposed in [4], which allows to optimize the set of projection angles based on certain prior knowledge about the object. In [5], an algorithm was proposed to select new projection angles based on the quantification of the projection information content using an entropy-like function of the already acquired projections. For tomography of elliptical objects, a genetic algorithm was proposed [6], which exploits the preferential direction characteristic of the objects and uses reconstructions from available projections to select the next projection directions. In [7], a new strategy was recently proposed for angle selection in binary tomography, which is based on the concept of information gain from adding a particular projection angle to the set of projection directions and does not require specifying prior knowledge about the object.

In the present paper, the dynamic angle selection strategy for binary object scanning is adapted for use in grey scale tomography. It is a dynamic algorithm, which selects a new angle based on the currently available projection data and incorporates two major concepts: (1) sampling of the set of images that are consistent with the already acquired projection data and (2) determining the amount of information that can be gained by acquiring a projection from a particular angle.

The structure of this paper is as follows. In Section 2 our approach is explained. Section 3 describes experiment setups and presents obtained results. The approach is discussed in Section 4. Finally, conclusions are drawn in Section 5.

2. Method

2.1. Information gain

The idea of the proposed angle selection algorithm is to select a new projection direction in such a way that the newly obtained projection will contain as much information about the object as possible. As a measure of information, a concept of information

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gain is used, which is based on the diameter of the set of solutions that are consistent with already obtained projections [7].

Let $\Theta = \{\theta_1, \ldots, \theta_d\}$be the current set of d angles, for which projection data $p^\Theta = W^\Theta v$ of the unknown image $v \in [0,1]^n$ have already been measured, where n is the number of pixels in the image and W^Θ is the projection matrix corresponding to Θ. Note that if the assumption on the range of the grey values of the unknown image is not satisfied in practice, a preprocessing step is needed to make the assumption valid, which is discussed in Section 4. Let $S_{W^\Theta}(p^\Theta) = \left\{x \in [0,1]^n : W^\Theta x = p^\Theta\right\}$ be the set of all solutions that are consistent with the projection data p^Θ. Then, the *information gain* for any image x and set of angles Θ yielded by taking a projection from angle θ is defined by

$$G(x, \Theta, \theta) = diam\left(S_{W^\Theta}\left(W^\Theta x\right)\right) - diam\left(S_{W^{\Theta\cup\{\theta\}}}\left(W^{\Theta\cup\{\theta\}} x\right)\right), \quad (1)$$

where $diam(V) = \max\left\{\|x-y\|_2 | x, y \in V\right\}$ for any $V \subset [0,1]^n$. This defines the information gain as the difference of the diameters of the sets of all images having the same projections as x. Having defined the mean information gain of a set of images $V \subset [0,1]^n$ as

$$G(V, \Theta, \theta) = \sqrt{\frac{\int_V G(x, \Theta, \theta)^2 \, dx}{\int_V dx}}, \quad (2)$$

the next projection angle can be found as

$$\theta_{d+1} = \arg\max_{\theta \in [0,\pi)} G(S_{W^\Theta}(p^\Theta), \Theta, \theta). \quad (3)$$

In practice, the integrals in Eq. (2) have to be approximated. In the present paper, three approximation steps are proposed. Firstly, the diameter of the solution set in Eq. (1) is substituted with its upper bound which was proposed in [8,9] for binary solutions and which also holds for the solutions belonging to $[0,1]^n$. Secondly, integration in Eq. (2) is replaced by a summation over a set of *surrogate solutions*, which represent the true set $S_{W^\Theta}(p^\Theta)$. Finally, the continuous domain of the candidate angle θ_{d+1} is substituted with a finite set of candidate angles, updated every time a new angle is chosen. These steps are explained in detail in the following sections.

2.2. Upper bound for the diameter of the solution set

The upper bound for the diameter used in Eq. (1) is based on the concept of the *central reconstruction* x^*, which is the shortest solution in $S_{W^\Theta}(p^\Theta)$, in the Euclidean sense. This reconstruction can be computed using the Conjugate Gradient Least Squares (CGLS) method, an iterative Krylov subspace method ([10]). Define the *central radius* by $R = \sqrt{\frac{\|p^\Theta\|_1}{d} - \|x^*\|_2^2}$. Then, the following theorem holds:

Theorem. *Let* $x, y \in S_{W^\Theta}(p^\Theta)$. *Then* $\|x-y\|_2 \le 2R$.

The proof of this theorem can be found in Appendix A.

2.3. Surrogate solutions

In order to evaluate the mean information gain defined by Eq. (2), the fraction under the square root is replaced by the mean information gain over the set of surrogate solutions, which are used as samples representing the true solution set $S_{W^\Theta}(p^\Theta)$. A surrogate solution is calculated from a template image, a randomly generated member of a given parameterized family of images. This template image is then used as a starting point for the Simultaneous Iterative Reconstruction Technique (SIRT) [11] that computes the surrogate solution consistent with already obtained projection data p^Θ. As the system $W^\Theta x = p^\Theta$ is severely underdetermined, the surrogate solution partly retains the features present in the template image. Hence, allowing sufficient variation within the set of template images results in variation of the surrogate solutions obtained and allows to control the approximation of the true solution set $S_{W^\Theta}(p^\Theta)$.

2.4. Candidate angles

An adaptive approach is proposed to modify the set of angles being considered at the subsequent angle selection steps. Let $A_{d+1} = \{\alpha_1, \alpha_2, \ldots, \alpha_l\},\ 0 \leqslant \alpha_1 \leqslant \alpha_2 \leqslant \ldots \leqslant \alpha_l < \pi$ be the set of candidate angles for selecting the next angle θ_{d+1}. Suppose that α_i is the best angle and $\theta_{d+1} := \alpha_i$. Then, let the candidate angle set used for the selection of the angle θ_{d+2} be defined as $A_{d+2} = \left\{\alpha_1, \ldots, \alpha_{i-1}, \frac{\alpha_{i-1}+\alpha_i}{2}, \frac{\alpha_i+\alpha_{i+1}}{2}, \alpha_{i+1}, \ldots, \alpha_l\right\}$. This procedure allows to better sample the candidate angle space near the angles which are likely to reveal more details in the object and still leaves the possibility for choosing completely new directions.

2.5. Dynamic angle selection algorithm

Combining all the approximation steps, an algorithm for estimating the mean information gain for a candidate angle can be defined (Algorithm 1). Based on this algorithm, the proposed angle selection approach iterates over the set of candidate angles and chooses the angle yielding the highest mean information gain.

Algorithm 1. Computing the mean information gain for a candidate angle θ, based on K surrogate solutions

```
Input: Θ = {θ_1, ..., θ_d}, p^Θ, θ
x* = CGLS(W^Θ, p^Θ); // compute central reconstruction
D = 2 sqrt(||p^Θ||_1 / d − ||x*||_2^2); // compute the upper bound of the solution
    set diameter
Φ = Θ ∪ {θ}; // includethe candidate angle into the set of
    projection angles
for i := 1 to K do // loop over K surrogate solutions
    x̃ = generateSurrogate(Θ, p^Θ);
    p̃^Φ = W^Φ x̃; //calculate new projection data that includes the
    projection for the candidate angle
    x̃* = CGLS(W^Φ, p̃^Φ); // compute the new shortest solution
    D̃_i = 2 sqrt(||p̃^Φ||_1 / (d+1) − ||x̃*||_2^2); // compute the new upper bound
end
Output: G = sqrt((1/K) Σ_{i=1}^{K} (D − D̃_i)^2) // the mean information gain
```

3. Experiments

Simulation experiments were run to assess the ability of the proposed algorithm to select favourable projection angles. The size of the phantoms was limited to 128 × 128 pixels due to the computational complexity of the approach. A quantitative evaluation

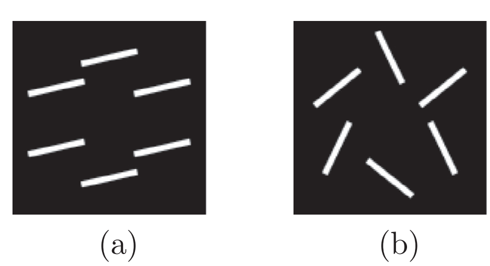

Fig. 1. Examples of the phantoms with one (a) and four (b) orientations, used in the experiments of Section 3.1.