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A Novel Empirical Equation for Relative Permeability in Low Permeability Reservoirs ☆

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ABSTRACT

In this paper, a novel empirical equation is proposed to calculate the relative permeability of low permeability reservoir. An improved item is introduced on the basis of Rose empirical formula and Al-Fattah empirical formula, with one simple model to describe oil/water relative permeability. The position displacement idea of bare bones particle swarm optimization is applied to change the mutation operator to improve the RNA genetic algorithm. The parameters of the new empirical equation are optimized with the hybrid RNA genetic algorithm (HRGA) based on the experimental data. The data is obtained from a typical low permeability reservoir well 54 core 27-1 in GuDong by unsteady method. We carry out matlab programming simulation with HRGA. The comparison and error analysis show that the empirical equation proposed is more accurate than the Rose empirical formula and the exponential model. The generalization of the empirical equation is also verified.

1. Introduction

Low permeability reservoirs are widespread in China continent. Among the new found reservoirs, the low and special low permeability reservoirs account for more than 70% [1]. The developing of this kind oilfield is the major problems we face at present as well as in the future. The relative permeability is an important parameter in improving oil production, increasing economic benefit, analyzing water production of oil wells, determining the vertical distribution of oil and water in reservoir, *etc* [2]. It is essential to have accurate relative permeability in engineering practice.

For calculation of relative permeability, Darcy formula is widely adopted in general reservoir [3]. For low permeability reservoirs, some influencing factors, such as start-up pressure gradient, capillary force, gravity and water saturation, play significant roles. The computation results with Darcy formula present large errors, so this mechanism model is not applicable [4].

Many researchers have computed relative permeability in low permeability reservoirs. Some people improved the Darcy formula. Song improved the Darcy formula with the influence of start-up pressure gradient on the basis of mass conservation equation [5]. Dong improved the Darcy formula with the influence of start-up pressure gradient through researching the oil-water movement equation and continuity equation [6]. Deng improved the Darcy formula with the influence of start-up pressure gradient, capillary force and gravity [7]. But these methods are tedious in computation and difficult to apply to engineering practice. Some researchers obtained the model of relative permeability in low permeability reservoirs with experimental data based on system identification. Honarpour *et al.* obtained the general model [8]. Al-Fatta obtained a model by linear and nonlinear multiple least-squares regression [9]. Ke got an exponential model by simulated annealing method [10,11].

The hybrid RNA genetic algorithm (HRGA) is from the development of biological science and technology. The structure and information of RNA molecule are well known. A genetic algorithm based on coding and biological molecular operation is widely concerned [12]. This method improves the search efficiency and optimization performance by coding individuals to biological molecules with bases [13,14]. Appropriate mutation operator can improve the population diversity and prevent premature. The mutation operator of classical RNA genetic algorithm (RGA) is fixed [15], so a suitable method to determine the mutation operator is needed. Kennedy have presented some improvement on particle swarm optimization (PSO) and proposed the bare bones particle swarm algorithm [16].

In the HRGA, the position displacement idea of bare bones PSO is applied to change the mutation operator. The nucleotide base encoding, RNA recoding and protein folding are reserved in the algorithm. Thus the strong global search capability is kept. To make sure the directivity of local searching, the optimal experience of whole population and the

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historical experience of individuals are used. The convergence speed and solution precision are improved [17].

In this paper, a new empirical equation for water/oil relative permeability in low permeability reservoirs is proposed. The equation is based on Rose empirical formula and Al-Fattah empirical equation, with an improved item introduced. After the preprocessing of experimental data with the improved Darcy formula proposed by Deng *et al.* [7], we use the HRGA to optimize the parameters in the new empirical equation. Then we compare this new empirical equation with Rose empirical formula and exponential model.

2. Statement of Problem

Some models have been proposed for estimation of relative permeability, but large deviations exist. To find an accurate empirical model, we first discuss the empirical equations of Rose and Al-Fattah.

Table 1 shows Rose empirical formula, where K_{rw} and K_{ro} are relative permeability of water and oil respectively. Its main advantage is that the two phase residual saturation must be known and quite accurate, so its application is limited. We will improve this method and use a general equation to calculate relative permeability.

Table 1
Rose empirical formula for calculating relative permeability

Rock type	K_{ro}	K_{rw}
Unconsolidated sand, good separation	$(1-S'_w)^3$	$(S'_w)^3$
Unconsolidated sand, bad separation	$(1-S'_w)^2(1-S'^{1.5}_w)$	$(S'_w)^{3.5}$
Cementation sandstone, limestone	$(1-S'_w)^2(1-S'^2_w)$	$(S'_w)^4$

The formula can be written as $K_{ro}=(1-S'_w)^m f(S'_w)$, where $S'_w=\frac{S_w-S_{wc}}{1-S_{wc}}$, so we have

$$K_{ro}=(1-S_w)^m f(S_w)/(1-S_{wc})^m \tag{1}$$

where m is an arbitrary constant, S_{wc} is irreducible water saturation and S_w is water saturation.

Al-Fattah established an accurate empirical equation in 2004 [9], the equation can be expressed as

$$K_{ro}=K_{ro}(S_{wc})\left(\frac{1-S_w}{1-S_{wc}}\right)^{3.661763}\left(\frac{1-S_w-S_{or}}{1-S_{wc}-S_{or}}\right)^{0.7} \tag{2}$$

where S_{or} denotes the residual oil saturation.

Eq. (2) can be written as

$$K_{ro}=\frac{K_{ro}(S_{wc})}{(1-S_{wc})^{3.661763}(1-S_{wc}-S_{or})^{0.7}}(1-S_w)^{3.661763}\cdot(1-S_w-S_{or})^{0.7} \tag{3}$$

Based on Eqs. (1) and (3), we propose a new empirical equation. An improved item defined as $f(S_w)=(S_w-C)^D$ is introduced. Replacing constant terms by A, B, C and D, we have a general empirical equation.

$$K_{ro}=A(1-S_w)^B(S_w-C)^D \tag{4}$$

For water phase, we have $K_{rw}=(S'_w)^n=(-1)^{n-B'}\frac{(1-S_w)^{B'}(S_w-C')^{n-B'}}{(1-S_{wc})^n}$, where $C'=1$.

With constant A', B' and D', this equation change to

$$K_{rw}=A'(1-S_w)^{B'}(S_w-C')^{D'} \tag{5}$$

We obtain a new general empirical equation to compute relative permeability of water and oil in low permeability reservoir,

$$K_{rw,ro}=A\cdot(1-S_w)^B\cdot(S_w-C)^D \tag{6}$$

The values of A, B, C and D are to be determined.

3. Computation Method of Relative Premeability Based on HRGA

The genetic algorithm is easy to realize and needs little knowledge to guide searching and especially suitable to solve complicated nonlinear optimization problem. In this paper, RNA genetic algorithm is adopted to optimize the parameters of empirical equation. However, RNA genetic algorithm has some flaws, such as slow convergence speed, early maturity and non-directional genetic operator. In order to overcome these shortcomings, the position displacement idea of bare bones PSO is applied to change the mutation operator to improve the genetic algorithm, which is the HRGA.

3.1. HRGA

Particle swarm algorithm is an optimization method on the basis of population, with the main idea from the simulation of flock foraging behavior. Each particle in the algorithm is a solution in the solution space. The position and speed are updated by population experience and individual optimization process, and the optimal solution is searched in multi-dimensional space [18,19].

Letting population size be N and particle dimension be m, we have the position of particle i in generation t, $X_i(t)=(x_{i1}(t),\cdots,x_{ij}(t),\cdots,x_{im}(t))$, $i=1,2,\cdots,N$; speed of particle i in generation t, $V_i(t)=(v_{i1}(t),\cdots,v_{ij}(t),\cdots,v_{im}(t))$; individual history optimal value of particle, $\mathrm{PBest}_i(t)=(\mathrm{pbest}_{i1}(t),\cdots,\mathrm{pbest}_{ij}(t),\cdots,\mathrm{pbest}_{im}(t))$ and the global optimal value of particle, $\mathrm{GBest}(t)=(\mathrm{gbest}_1(t),\cdots,\mathrm{gbest}_j(t)\cdots,\mathrm{gbest}_m(t))$.

Then particle updates the position and speed as follows,

$$\begin{aligned}v_{ij}(t+1)&=wv_{ij}(t)+c_1\cdot r_{1j}\cdot\left(\mathrm{pbest}_{ij}(t)-x_{ij}(t)\right)+c_2\cdot r_{2j}\cdot\left(\mathrm{gbest}_j-x_{ij}(t)\right)\\x_{ij}(t+1)&=x_{ij}(t)+v_{ij}(t+1)\end{aligned} \tag{7}$$

In the bare bones particle swarm algorithm proposed by Kennedy [20], following expressing is used as the evolution equation of standard particle swarm algorithm instead of Eq. (7).

$$x_{ij}(t+1)=N\left(\frac{\mathrm{pbest}_{ij}(t)+\mathrm{gbest}_j(t)}{2}\left|\mathrm{pbest}_{ij}(t)-\mathrm{gbest}_j(t)\right|\right) \tag{8}$$

The positions of particles are random number with Gauss distribution with the mean value of $\frac{\mathrm{pbest}_{ij}(t)+\mathrm{gbest}_j(t)}{2}$ and standard deviation of $|\mathrm{pbest}_{ij}(t)-\mathrm{gbest}_j(t)|$.

RNA genetic algorithm is on the basis of base coding and biological molecule operation. In a biological molecule, every three bases compose one amino acid, *i.e.*, the length of bases is divided by 3. With RNA recoding and protein folding [21], to reduce calculation and control population size, we assume that the protein folding operation only occurs on individuals without RNA recoding. Then the important work is to change the mutation probability [22,23].

The essence of position updating of particle swarm is a mutation operation [24]. Traditional RNA genetic algorithm mutates at the fixed mutation probability, while HRGA can reflect the historic information of individuals and the sharing information of the population. HRGA can make every individual do directional mutation and improve search efficiency. Moreover, HRGA ensures strong global search capability since it does not change the selection and crossover operator. In HRGA, we use Eq. (8) as the mutation operator to improve RNA genetic algorithm.