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Fluorescence of polarized atoms excited by polarized electrons

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ABSTRACT

A general expression for the cross section describing properties of the fluorescence radiation from polarized atoms excited by polarized electrons is obtained in the distorted wave (DW) approximation. The expression is presented in the form of multiple expansion over the state multipoles of all particles taking part in the two-step process. It is used to derive the alignment parameters of excited atom and angular distribution of emitted fluorescence. The polarization degree of radiation from the lowest autoionizing states $np^5(n+1)\ s^2\ {}^2P_{3/2}$ following the excitation of non-polarized Na and K atoms by non-polarized electrons is calculated.

1. Introduction

The radiation emission is an important source of information about the processes taking part in laboratory and astrophysical plasmas. The simulated spectra of intensities and polarization are used for plasma diagnostics, e.g. tokamak [1]. The excitation of atoms by electrons is one of the most important processes leading to the fluorescence emission [2]. The collimated beam of electrons introduces a large degree of anisotropy into the atomic system. The alignment is manifested in the non-uniform angular distribution of radiation [3]. The use of the polarized atoms and electrons in the experiment can provide further information about the structure of atoms and the dynamics of the process. Therefore, general expressions for the cross sections involve description of the polarization state of all particles both in initial and final states are of importance. The main task of this work is to present such general expression for differential cross section describing the fluorescence following excitation of polarized atoms by polarized electrons in non-relativistic approximation.

To derive expressions for the parameters suitable to characterize the polarization state of the particles participating in the electron–atom interaction, a density matrix formalism has been widely used [4], but also a number of alternative approaches was proposed [5–8]. In our method the cross section is expressed as a multiple expansion over the multipoles of the states of all particles taking part in the process instead of using the density matrix elements. The most general cases with respect to the number of polarized states were investigated. So far, the method has been applied to obtain a general expression for fluorescence radiation following recombination of polarized ions with polarized electrons [9], radiative [10] and Auger decay [11] following photoionization of polarized atoms, excitation [12,13] and ionization [14,15] of polarized atoms by polarized electrons. The method was also generalized for multi-step processes [8].

2. General expression

The process of the radiation characterized by the wave vector $\mathbf{k}_{01}$ and polarization unit vector $\hat{\epsilon}_q$ which follows the excitation of polarized atom by polarized electron can be written as follows:

$$A(\alpha_0 J_0 M_0) + e(\mathbf{p}_1 m_1) \rightarrow A^*(\alpha_1 J_1) + e(\mathbf{p}_2 m_2) \rightarrow A(\alpha_2 J_2 M_2) + e(\mathbf{p}_2 m_2) + h\nu(\hat{\epsilon}_q \mathbf{k}_{01}). \quad (1)$$

Here α_i denotes the configuration and other quantum numbers, while J_i and M_i are the total angular momentum and its projection onto the chosen direction, respectively, for atoms in the initial ($i=0$), intermediate ($i=1$) and final ($i=2$) states, $\mathbf{p}_i m_i$ denote the electron momentum and spin projection.

In two-step approximation, the expression of the differential cross section describing fluorescence radiation following the excitation of polarized atoms by polarized electrons can be written in the form of the expansion over the multipoles of the

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non-registered intermediate state of an excited atom $A(\alpha_1 J_1 M_1)$ by using the method proposed in [8] as follows:

$$\frac{d^2\sigma(\alpha_0 J_0 M_0 \mathbf{p}_1 m_1 \to \alpha_1 J_1 \mathbf{p}_2 m_2 \to \alpha_2 J_2 M_2 \hat{\epsilon}_q \mathbf{k}_{01})}{d\Omega_e d\Omega_f} = \sum_{K_1 N_1} \frac{d\sigma^{\mathrm{ex}}_{K_1 N_1}(\alpha_0 J_0 M_0 \mathbf{p}_1 m_1 \to \alpha_1 J_1 \mathbf{p}_2 m_2)}{d\Omega_e} \times \frac{dW^r_{K_1 N_1}(\alpha_1 J_1 \to \alpha_2 J_2 M_2 \hat{\epsilon}_q \mathbf{k}_{01})}{d\Omega_f}. \quad (2)$$

Here $d\Omega_e$ and $d\Omega_f$ indicate solid angles of the scattered electron and emitted photon, respectively. The range of the summation over K_1 depends on the value of the total angular momentum J_1 and on second process. In the case of electric dipole radiation and $J_1 \geqslant 1$, it follows that $K_1 = 0, 1, 2$. The expressions for the first and second factor above are presented in [12] and [9]. They are given by

$$\frac{d\sigma^{\mathrm{ex}}_{K_1 N_1}(\alpha_0 J_0 M_0 \mathbf{p}_1 m_1 \to \alpha_1 J_1 \mathbf{p}_2 m_2)}{d\Omega} = 4\pi C [2K_1+1]^{1/2} \times \sum_{\substack{K, K_0, K_0', K_{\lambda 1}, K_{s1} \\ K_1', K_{\lambda 2}, K_{s2}}} \mathscr{B}^{\mathrm{ex}}(K_0, K_0', K_1, K_1', K_{\lambda 1}, K_{s1}, K_{\lambda 2}, K_{s2}, K) \times \sum_{\substack{N_0, N_0', N_{\lambda 1}, N_{s1} \\ N_1', N_{\lambda 2}, N_{s2}, N}} \begin{bmatrix} K_{\lambda 1} & K_{s1} & K_0' \\ N_{\lambda 1} & N_{s1} & N_0' \end{bmatrix} \begin{bmatrix} K_0 & K_0' & K \\ N_0 & N_0' & N \end{bmatrix} \begin{bmatrix} K_1 & K_1' & K \\ N_1 & N_1' & N \end{bmatrix} \times \begin{bmatrix} K_{\lambda 2} & K_{s2} & K_1' \\ N_{\lambda 2} & N_{s2} & N_1' \end{bmatrix} Y^*_{K_{\lambda 1} N_{\lambda 1}}(\hat{p}_1) Y_{K_{\lambda 2} N_{\lambda 2}}(\hat{p}_2) T^{*K_0}_{N_0}(J_0, J_0, M_0 | \hat{J}_0) \times T^{*K_{s1}}_{N_{s1}}(s, s, m_0 | \hat{s}) \times T^{K_{s2}}_{N_{s2}}(s, s, m_1 | \hat{s}), \quad (3)$$

$$\frac{dW^r_{K_1 N_1}(\alpha_1 J_1 \to \alpha_2 J_2 M_2 \hat{\epsilon}_q \mathbf{k}_0)}{d\Omega_f} = C_1 \sum_{K_r, K_2, k, k'} \mathscr{A}(K_1, K_r, K_2, k, k') \times \sum_{N_r, N_2} \begin{bmatrix} K_1 & K_r & K_2 \\ N_1 & N_r & N_2 \end{bmatrix} T^{K_2}_{N_2}(J_2, J_2, M_2 | \hat{J}_2) T^{K_r}_{N_r}(k, k', q | \hat{\mathbf{k}}_0). \quad (4)$$

The constant in (3) is $C = 4/p_1^2$ ($p_1 = \sqrt{2\varepsilon_1}$ in atomic units), $C_1 = 1/(2\pi)$, $s = 1/2$ is the spin of an electron, $Y_{KN}(\hat{J})$ is the spherical function, the hat denotes the polar and azimuthal angles with respect to the chosen z axis, the square brackets stand for the Clebsh–Gordan coefficient, and

$$T^K_N(J, J, M | \hat{J}) = (-1)^{J_0 - M_0} \left[\frac{4\pi}{2J+1}\right]^{1/2} \begin{bmatrix} J & J & K \\ M & -M & 0 \end{bmatrix} Y_{KN}(\hat{J}). \quad (5)$$

The expressions for the parameters $\mathscr{B}^{\mathrm{ex}}$ and $\mathscr{A}$ in (3) and (4) are as follows [12,9]:

$$\begin{aligned} \mathscr{B}^{\mathrm{ex}} = & \sum_{\lambda_1, \lambda_1', \lambda_2, \lambda_2', j_1, j_1', j_2, j_2', J, J'} (2J+1)(2J'+1)(2s+1)(-1)^{\lambda_1' + \lambda_2'} \\ & \times \langle \alpha_1 J_1, \varepsilon_2 \lambda_2 (j_2) J \| V \| \alpha_0 J_0, \varepsilon_1 \lambda_1 (j_1) J \rangle \langle \alpha_1 J_1, \varepsilon_2 \lambda_2' (j_2') J' \| V \| \alpha_0 J_0, \varepsilon_1 \lambda_1' (j_1') J' \rangle^* \\ & \times [(2\lambda_1+1)(2\lambda_1'+1)(2\lambda_2+1)(2\lambda_2'+1)(2j_1+1)(2j_1'+1) \\ & \times (2j_2+1)(2j_2'+1) \times (2J_0+1)(2J_1+1)(2K_0'+1)(2K_1'+1)]^{1/2} \\ & \times \begin{bmatrix} \lambda_1 & \lambda_1' & K_{\lambda 1} \\ 0 & 0 & 0 \end{bmatrix} \\ & \times \begin{bmatrix} \lambda_2 & \lambda_2' & K_{\lambda 2} \\ 0 & 0 & 0 \end{bmatrix} \times \begin{Bmatrix} J_0 & K_0 & J_0 \\ j_1' & K_0' & j_1 \\ J' & K & J \end{Bmatrix} \begin{Bmatrix} \lambda_1' & K_{\lambda 1} & \lambda_1 \\ s & K_{s1} & s \\ j_1' & K_0' & j_1 \end{Bmatrix} \\ & \times \begin{Bmatrix} \lambda_2' & K_{\lambda 2} & \lambda_2 \\ s & K_{s2} & s \\ j_2' & K_1' & j_2 \end{Bmatrix} \begin{Bmatrix} J_1 & K_1 & J_1 \\ j_2' & K_1' & j_2 \\ J' & K & J \end{Bmatrix}, \end{aligned} \quad (6)$$

$$\mathscr{A} = (\alpha_2 J_2 \| Q^{(k)} \| \alpha_1 J_1)(\alpha_2 J_2 \| Q^{(k')} \| \alpha_1 J_1)^* \times \left[\frac{(2K_1+1)(2J_2+1)(2k+1)}{2K_2+1}\right]^{1/2} \begin{Bmatrix} J_1 & K_1 & J_1 \\ k & K_r' & k' \\ J_2 & K_2 & J_2 \end{Bmatrix}. \quad (7)$$

In (6) and (7), the braces denote 9j-coefficient. The reduced matrix element in (7) is expressed [10]:

$$\langle \alpha_2 J_2 \| Q^{(k)} \| \alpha_1 J_1 \rangle = k_0^{k-1/2} \sum_{p=0,1} \left[\frac{k+1}{k}\right]^{1/2} \frac{i^k}{(2k-1)!!} \langle \alpha_2 J_2 | \mathscr{D}^p_k | \alpha_1 J_1 \rangle. \quad (8)$$

For the electrical multipole (Ek) transitions, $p = 0$, and the transition operator in (8) is [10]

$$\mathscr{D}^0_{kq} = -r^k C^{(k)}_q \quad (9)$$

and, for the magnetic multipole transition (Mk)($p = 1$), it is

$$\mathscr{D}^1_{kq} = \frac{iq}{c} [k(2k-1)]^{1/2} r^{k-1} \left\{ \frac{1}{k+1} \left[C^{(k-1)} \times L^{(1)} \right]^{(k)}_q + \left[C^{(k-1)} \times S^{(1)} \right]^{(k)}_q \right\}. \quad (10)$$

Here L and S are the operators of the orbital and spin angular momentum, respectively, $C^{(k)}_q$ is the operator of the spherical function normalized to $[4\pi/(2k+1)]^{1/2}$.

The expression (2) together with (3), (4), (6) and (7) for the two-step process (1) is very general. It describes the polarization state of all particles taking part in the process and the angular distribution of the reaction products. It can be used for the derivation of some special cases applicable for specific experimental conditions with smaller number of polarization states specified.

3. Applications

In the case of electric dipole electromagnetic radiation ($k = k' = 1$), the cross section describing angular distribution of linearly polarized radiation which follows excitation of non-polarized atoms by non-polarized electrons can easily be obtained from the general expression (2) ($K_0 = N_0 = K_{s1} = N_{s1} = K_{\lambda_2} = N_{\lambda_2} = K_{s2} = N_{s2} = K_2 = N_2 = N_{\lambda_1} = N_1 = 0$, $K_1 = K_r = 2$):

$$\frac{\sigma(\alpha_0 J_0 \to \alpha_1 J_1 \to \alpha_2 J_2 \hat{\epsilon}_q \mathbf{k}_{01})}{d\Omega_f} = \frac{1}{2(2J_0+1)} \times \int d\Omega_e \sum_{M_0, M_2, m_1, m_2} \frac{d^2\sigma(\alpha_0 J_0 M_0 \mathbf{p}_1 m_1 \to \alpha_1 J_1 \mathbf{p}_2 m_2 \to \alpha_2 J_2 M_2 \hat{\epsilon}_q \mathbf{k}_{01})}{d\Omega_e d\Omega_f} = \frac{\sigma(\alpha_0 J_0 \to \alpha_1 J_1 \to \alpha_2 J_2)}{4\pi} [1 + \beta P_2(\cos\theta)]. \quad (11)$$

Here the laboratory z axis coincides with the direction of the electron projectile, and θ is the angle of emitted radiation with respect to the same axis, and $P_2(\cos\theta)$ is the Legendre polynomial. In (11),

$$\beta = A_2 \alpha^r \quad (12)$$

is the asymmetry parameter of the angular distribution of emitted fluorescence. Here the first factor is the alignment of the non-polarized atoms excited by non-polarized electrons [12] along the direction of projectile electrons

$$A_2 = \frac{5\mathscr{B}^{\mathrm{ex}}(0,2,2,0,2,0,0,0,2)}{\mathscr{B}^{\mathrm{ex}}(0,0,0,0,0,0,0,0,0)}, \quad (13)$$

and the second factor describes the asymmetry of spontaneous decay of the excited state of the atom

$$\alpha^r = \left[\frac{3(2J_1+1)}{2}\right]^{1/2} (-1)^{J_1+J_2+1} \begin{Bmatrix} 1 & 1 & 2 \\ J_1 & J_1 & J_2 \end{Bmatrix}, \quad (14)$$

where the braces $\{\ldots\}$ denote the 6j-coefficient.

In the case of linearly polarized electric dipole radiation, the polarization degree is defined as follows: $P = 3\beta/(\beta - 2)$.