Available online at www.sciencedirect.com
SCIENCE DIRECT®
Nuclear Instruments and Methods in Physics Research B 247 (2006) 138–142
NIM B
Beam Interactions with Materials & Atoms
www.elsevier.com/locate/nimb

Low-temperature antihydrogen-atom scattering

S. Jonsell *

Department of Physics, Umeå University, SE-90187 Umeå, Sweden

Available online 9 March 2006

Abstract

A simple method to include the strong force in atom–antiatom scattering is presented. It is based on the strong-force scattering length between the nucleon and antinucleon. Using this method elastic and annihilation cross-sections are calculated for hydrogen–antihydrogen and helium–antihydrogen scattering. The results are compared to first-order perturbation theory using a pseudo-potential. The pseudo-potential approach works fairly well for hydrogen–antihydrogen scattering, but fails for helium–antihydrogen scattering where strong-force effects are more prominent.

PACS: 36.10.−k; 34.50.−s

Keywords: Antihydrogen; Antimatter; Scattering; Annihilation

Antimatter in contact with matter causes annihilation – a violent process where mass is converted to energy. This is well known, even among people who never have studied physics antimatter annihilation seems to stimulate the imagination. In popular films and literature antimatter is used as e.g. an energy source, fuel for space ships, or a weapon. However, since the energy required to create antimatter outweighs the energy created by annihilation by many orders of magnitude, neither of these applications are possible in reality. More realistically, antimatter may have applications in medicine, such as antiproton treatment of cancer.

The primary goal for antimatter research is not futuristic applications, but to uncover the symmetries (or possible asymmetries) of the fundamental laws of physics. One could expect that the matter and antimatter contents of the universe would be symmetric, but so far observations indicate that at least in the neighborhood of our galaxy there is only matter. Today the coexistence of matter and antimatter is ruled out on scales $\lesssim 20$ Mpc [1]. The reason for this matter–antimatter asymmetry could be a violation of the CPT theorem. This theorem, and other matter–antimatter symmetries, could be tested through high-precision measurements on antihydrogen. Two experiments at CERN, ATHENA and ATRAP, have succeeded in creating antihydrogen atoms, but much work remains before CPT tests can be carried out [3].

Despite the interests in antimatter of both scientists and the general public, relatively little is known about the interaction between atoms made of antimatter and ordinary atoms. So far no experimental data exist, and theoretical work has only just started. Atom–antiatom scattering has practical implications for antihydrogen experiments, e.g. unwanted scattering of antiatoms on impurities or on the walls of the experiment, or scattering on ultracold atoms introduced on purpose to achieve sympathetic cooling of antimatter. Atom–antiatom scattering is also interesting in its own right, being a new fundamental process involving both atomic and low-energy nuclear physics.

A number of inelastic processes are possible in atom–antiatom scattering: nucleus–antinucleus annihilation, electron–positron annihilation, rearrangement processes (such as formation of positronium) and even formation of metastable "molecules". The most important of these are nucleus–antinucleus annihilation and rearrangement.

* Tel.: +46 90 7867897; fax: +46 90 7866673.
E-mail address: jonsell@tp.umu.se

0168-583X/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.nimb.2006.01.050

Rearrangement processes will also eventually lead to annihilation, but on a time scale much longer than the time required for the final-state fragments to separate. Hence, it can be regarded as a process distinct from the "direct" annihilation during the collision. Alternatively, rearrangement can be regarded as a resonance in the annihilation channel. In this paper, I will focus on the direct annihilation process, while rearrangement is discussed extensively elsewhere in these proceedings [4].

At large internuclear separations R the wave function of the atom–antiatom system with relative angular momentum l is given by

$$\Phi(\boldsymbol{R},\boldsymbol{r}_i) \to N\frac{1}{R}\sin(k_iR - l\pi/2 + \delta_l)Y_{lm}(\Omega_R)\psi_{\overline{\mathrm{H}}}(\boldsymbol{r}_1)\phi_{\mathrm{atom}}(\boldsymbol{r}_{i>1}), \tag{1}$$

where $\psi_{\overline{\mathrm{H}}}$ and ϕ_{atom} are the atomic wave functions of antihydrogen and the atom, $\boldsymbol{r}_i$ denotes the coordinates of all leptons, and $\hbar k_i$ is the relative momentum in the initial state. Both elastic and annihilation cross-sections are given by the complex phase shift δ_l [2]

$$\sigma^{\mathrm{el}} = \frac{\pi}{k_i^2}(2l+1)|1-\mathrm{e}^{2\mathrm{i}\delta_l}|^2, \tag{2}$$

$$\sigma^{\mathrm{a}} = \frac{\pi}{k_i^2}(2l+1)(1-\mathrm{e}^{-4\mathrm{Im}\delta_l}). \tag{3}$$

At low temperatures ($\lesssim 1$ K) only the $l = 0$ partial wave contributes, and the phase shift has the low-energy form

$$\lim_{k_i\to 0} \tan\delta_0 = -k_i a, \tag{4}$$

where $a = \alpha - \mathrm{i}\beta$ is the complex scattering length. Inserting (4) into (2) and (3) gives the low-energy form of the cross-sections

$$\sigma^{\mathrm{el}} = 4\pi(\alpha^2+\beta^2), \tag{5}$$

$$\sigma^{\mathrm{a}} = \frac{4\pi}{k_i}\beta. \tag{6}$$

We first note that the elastic cross-section is constant at low energies, while the annihilation cross-section diverges as k_i^{-1}. Thus, below a certain energy annihilation dominates. Second, we note that σ^{el} depends on both α and β. This means that a non-zero imaginary part β of the scattering length, arising from the inclusion of the strong nuclear force, will not only give rise to annihilation, but will also significantly modify the elastic cross-section. Only if the annihilation cross-section is small, $\beta \ll |\alpha|$, this effect may be ignored.

The simplest way to calculate the annihilation cross-section is to use a pseudo-potential

$$V_a(\boldsymbol{R}) = A\delta(\boldsymbol{R}). \tag{7}$$

This is a contact interaction with strength given by the constant A. This constant can be determined from experimental data on the nucleus–antinucleus system, or it can be calculated from model potentials. Using the pseudo-potential in first-order perturbation theory the annihilation cross-section becomes

$$\sigma^{\mathrm{a}} = \frac{(2\pi)^3}{k_i^2}A\lim_{R\to 0}\int |\Phi(\boldsymbol{R},\boldsymbol{r}_i)/R|^2\,\mathrm{d}\boldsymbol{r}_i. \tag{8}$$

For the proton–antiproton system $A^{\mathrm{p\bar{p}}} = 1.7\times 10^{-7}$ a.u. $= 6.8\times 10^{-37}$ eV m^3, which was determined from the width 1130 eV of the 1s state of protonium [5]. For the alpha particle–antiproton system no data on the 1s state are available, instead the value $A^{\alpha\bar{\mathrm{p}}} = 3.4\times 10^{-7}$ a.u. $= 1.4\times 10^{-36}$ eV m^3 was determined from a combination of low-energy annihilation data, the energy shift of the 2p state, and the width of the 2p and 3d state [6]. Both values represent an average over different spin states.

The pseudo-potential approach, being based on first-order perturbation theory, does not take into account the modification of the initial channel due to the strong force. Even if annihilation cannot be treated perturbatively one may still use the zero-range approximation of the strong force, since the atomic nucleus is very small on an atomic scale. The detailed shape of the strong-force potential will therefore not matter, only its strength as parameterized by a short-range strong-force phase shift δ_{sf}. Furthermore, atomic energies (eV) are very small compared to typical nuclear energies (MeV). The strong-force phase shift can therefore be parameterized by a strong-force scattering length a_{sf}. This scattering length is complex, with the imaginary part representing annihilation.

The role of the strong-force scattering length can be understood by looking at the boundary condition of the wave function at short internuclear separations, $R \to 0$. In the absence of the strong force, the wave function has a short-distance form proportional to the regular Coulomb function for zero angular momentum $F_0(kR)$. Even when the strong force is added the wave function is Coulombic for $R_0 \leqslant R \ll 1a_0$, where R_0 is just outside the range of the strong force. But now, since the Coulombic solution is not extended all the way to $R = 0$, also the irregular Coulomb function $G_0(kR)$ is allowed. A general eigensolution to the zero-angular momentum Coulomb problem is an arbitrary linear combination of $F_0(kR)$ and $G_0(kR)$. For atom–antiatom scattering in the presence of the strong nuclear force the appropriate short-range form of the scattering function is given by a particular solution determined by the strong-force phase shift δ_{sf}

$$\Phi(\boldsymbol{R},\boldsymbol{r}_i) = \frac{F_0(kR)+\tan\delta_{\mathrm{sf}}G_0(kR)}{F_0(kR_0)+\tan\delta_{\mathrm{sf}}G_0(kR_0)}\varphi(\boldsymbol{r}_i). \tag{9}$$

Here $\varphi(\boldsymbol{r}_i)$ is the wave function of the leptons in the combined electric field of the overlapping nucleus and antinucleus. The wave vector k is related to k_i as $\hbar^2k^2/(2\mu) = \hbar^2k_i^2/(2\mu) - E_f$, where E_f is the leptonic energy corresponding to $\varphi(\boldsymbol{r}_i)$. Since R_0 is very small on an atomic scale, the boundary condition of Φ at R_0 may be expressed in terms of the short-range expansions of the Coulomb functions [8]

$$F_0(kR) \;\to\; C_0(\eta)kR, \tag{10}$$

$$G_0(kR) \;\to\; C_0^{-1}(\eta)\{1-2\eta kR[\ln(2\eta kR)+h(\eta)+2\gamma-1]\}. \tag{11}$$