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Propagation and generation of acoustic and entropy waves across a moving flame front

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ABSTRACT

In analytical models of the propagation and generation of acoustic and entropy waves across a premixed flame, the relations that couple upstream and downstream flow variables often consider the flame as a discontinuity at rest. This work shows how the model of a flame at rest can misrepresent the generation of entropy waves, and how it leads to paradoxical results concerning the conservation of mass and volume flow rates across the flame. Such inconsistencies can be resolved by taking into account the movement of the flame in the coupling relations for flow perturbations. Analysis in a quasi-1D framework shows that in the absence of perturbations in equivalence ratio, the magnitude of the entropy waves generated across the flame are *first order* in Mach number and derive from interactions between the upstream acoustics and the *mean* heat release rate. For non-perfectly premixed flames, fluctuations in equivalence ratio may generate perturbations in entropy of *leading order* in Mach number. Furthermore, for the moving flame model conservation of volume flow rate across a passive, perfectly premixed flame appears as a natural consequence of mass and energy conservation.

1. Introduction

In lean combustion systems, one of the major challenges to technological progress is thermo-acoustic instability. Such instabilities may be caused by fluctuations in pressure or velocity, i.e. acoustic disturbances, which impinge on the flame, causing the heat release rate to become unsteady. Fluctuations in heat release rate will in turn generate more acoustic disturbances, so that a feedback-loop is established, which may result in self-excited instability.

Acoustic perturbations at a flame can also cause so-called "entropy waves", i.e. temperature inhomogeneities in the burnt gases that are transported convectively. As Marble and Candel [1] have explained, when such inhomogeneities experience acceleration downstream of the flame (e.g. through a nozzle), acoustic waves are generated in both upstream and downstream directions from the zone of acceleration. The upstream propagating component travels back into the combustion chamber, contributing to the acoustic oscillations in the system. This mechanism can also trigger thermo-acoustic instabilities, see [2–5].

According to Rayleigh [6], instabilities in a thermo-acoustic system can occur, when thermal and acoustic disturbances interact constructively. Therefore, understanding the mechanisms of acoustic and entropy waves generation across the flame, and their propagation in the system is crucial for prediction and control of the thermo-acoustic instabilities.

To predict system instabilities, the framework of low-order network models is widely employed [5,7–13]. In this framework, a one-dimensional thermo-acoustic system is represented as a network of acoustic elements, each one characterized by its transfer matrix, which expresses the relations between the flow perturbations in velocity u', pressure p' and entropy s' upstream of the element to the perturbations downstream, see [14].

In an idealized treatment, such relations may be derived analytically from the linearized conservation equations for mass, momentum and energy. The effect of a heat source on the acoustic field may also be deduced from these conservation equations. In this case, the analysis should describe the scattering of acoustic waves by the temperature and density gradients that result from mean heat release rate $\bar{\dot{Q}}$, as well as account for the coupling between the fluctuations of heat release $\dot{Q}'$ and the acoustic perturbations.[1]

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[1] Here and in the following, overbars $\bar{\ldots}$ denote mean values, while primed quantities $\ldots'$ refer to fluctuations around the mean.

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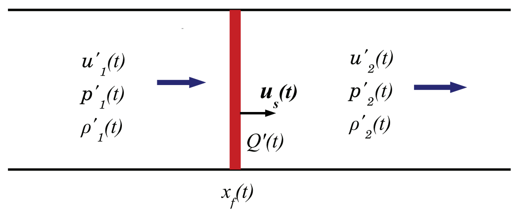

Fig. 1. A compact heat source in quasi-1D flow with flow perturbations u', p', ρ' at upstream ("1") and downstream ("2") locations, respectively. For a heat source at rest, the velocity of the heat source in the laboratory frame $u_s(t) = 0$, thus the location of the source, $\bar{x}_f$, is constant in time.

The configuration considered in the present paper is depicted in Fig. 1. The heat source is regarded as a one-dimensional discontinuity. This is appropriate if the heat source is *compact*, i.e. if both acoustic and entropy wavelengths are much larger than the axial extent of the heat source. Moreover, it is often assumed that the heat source is *fixed at position* $\bar{x}_f$. A derivation of thermo-acoustic coupling relations by analysis of the conservation laws for mass, momentum and energy as they apply for a compact heat source at rest can be found in several prior studies, see e.g., [10,15–17].

However, paradoxical conclusions may result from the thermo-acoustic coupling relations for a heat source at rest. The first contradiction concerns the production of entropy waves by a heat source. The coupling relations for a heat source at rest imply that in general unsteady heat release $\dot{Q}' \neq 0$ should result in the generation of entropy waves, i.e. $s' \neq 0$ downstream of the heat source, see [10,18,19]. However, in the case of a perfectly premixed flame with homogeneous fuel/air premixture, the presence of significant entropy waves (i.e. temperature inhomogeneities) downstream of the combustion zone is difficult to justify physically, because in the case of adiabatic and complete combustion, the temperature increase across the flame and thus also the temperature downstream of the flame should be constant.

The second issue has been raised by Bauerheim et al. [17] for the case of a passive flame ($\dot{Q}' = 0$) at rest, in the limit of vanishing mean flow Mach number. In the absence of mean flow, the energy conservation equation is reduced to conservation of volume flux, which implies that fluctuations of upstream ("1") and downstream ("2") velocities be equal, $u'_1 = u'_2$. However, this is in apparent contradiction with mass conservation, which for $M = 0$ would seem to impose $u'_1\bar{\rho}_1 = u'_2\bar{\rho}_2$. Bauerheim et al. [17] reexamined the quasi-1D conservation equations and observed that acoustic and entropy perturbations are coupled. At zero Mach number, a singularity in entropy is produced, which acts as an additional source term in the mass balance equation, which "explains why mass conservation of fluctuations is satisfied at non-zero Mach number while volume flow rate is conserved at zero Mach number" [17]. Thus the paradox is resolved, but the conclusions that result from the mathematical arguments are not easily reconciled with physical intuition.

For the two cases mentioned above, conclusions developed from linearized conservation equations for mass and energy are either apparently contradictory, or non-intuitive. This is rather unsatisfactory, since mathematical models should represent and clarify the actual physical problem. The physical meaning of the interdependency among entropy waves generation, unsteady heat release and mass flow conservation needs to be re-examined and contextualized by revisiting the coupling relations and the underlying assumptions.

In this work, it will be shown how the issues described can be resolved by relaxing the assumption that the heat source is at rest. Instead, the flame front will be considered as a moving discontinuity, which implies that movement of the heat source must be taken into account in the conservation equations. Equations which describe the propagation of small flow disturbances across a moving heat source were first derived by Chu [20,21]. Although the moving flame model has been used since in many studies, see e.g., [7,12,22–29], its consequences on acoustic scattering and generation of entropy wave have not been fully explored. The present paper will analyze these consequences, by verifying the validity of the equations with physical arguments and examples.

In Section 2, we will introduce the difference between a moving heat source and a heat source at rest and explicate some consequences of movement of the heat source. In particular, the linearized conservation equations for perturbations of velocity, pressure and entropy across a moving heat source are analyzed (Section 3). Section 4 turns to the particular case of a moving premixed flame front, with fluctuations in heat release rate, flame speed and flame surface area in response to upstream velocity perturbations. Next, the consequences of the flame front movement on entropy generation and acoustic scattering are examined for both perfectly and non-perfectly premixed flame (Sections 5 and 6). Finally, after terminology is established and the main results of this study are presented, Section 7 discusses and contextualizes previous publications on the model of a moving flame [7,12,22–29] and a flame at rest [10,15–17,30–32], respectively.

2. Motivation

In this section, we state the problems discussed in the previous section in mathematical terms and discuss some of the limitations that are implied with the application of the conservation equations of mass, momentum and energy to a heat source at rest. For the sake of simplicity, the case of a "passive source", i.e. a heat source without fluctuations of the heat release rate, $\dot{Q}' = 0$, will be considered.

In presence of perturbations, relevant variables are divided into a mean component, which varies spatially, and a fluctuating component, which in general is a function of both time and space:

$$\varphi(x,t) = \bar{\varphi}(x) + \varphi'(x,t). \tag{1}$$

For analysis of the perturbations across a compact heat source, a commonly adopted approach is to consider the linearized conservation equations just upstream and downstream of a discontinuity. The equations for conservation of mass, momentum and energy read (cf. [5,10,15–17,32,33]):

$$\begin{aligned}
&[\rho'\bar{u} + u'\bar{\rho}]_1^2 = 0,\\
&[p' + \rho'\bar{u}^2 + 2\bar{\rho}\bar{u}u']_1^2 = 0,\\
&[c_p\bar{T}(\rho'\bar{u} + u'\bar{\rho}) + \bar{\rho}\bar{u}(c_pT' + \bar{u}u')]_1^2 = \dot{Q}'.
\end{aligned} \tag{2}$$

Angular brackets $[\varphi]_1^2$ with sub-/super-scripts denote the difference between values of a flow variable φ upstream ("1") and downstream ("2") of the jump, i.e. $[\varphi]_1^2 = \varphi_2 - \varphi_1$. As mentioned in the previous section, the source region is considered as infinitesimally thin (i.e. compact with respect to acoustic and, in presence of mean flow, to entropy waves [10]) and fixed at position $\bar{x}_f$ in the stream-wise direction (see Fig. 1). The discontinuity is regarded as a "black-box", and its dynamic response to upstream perturbations is only represented by the source term $\dot{Q}'$.

In order to simplify the analysis, the fluctuating terms may be normalized by their respective mean values. Additionally, since flow regimes of interest are typically characterized by $M \ll 1$, terms of second or higher order in Mach number may be