Available online at www.sciencedirect.com
ScienceDirect
Nuclear Instruments and Methods in Physics Research B 266 (2008) 3715–3720
NIM B
Beam Interactions with Materials & Atoms
www.elsevier.com/locate/nimb

The features of synchrotron radiation from a relativistic particle rotating inside a spherical cavity

S.R. Arzumanyan, L.Sh. Grigoryan *, H.F. Khachatryan, M.L. Grigoryan

Institute of Applied Problems in Physics, 25 Hr. Nersessian Street, 0014 Yerevan, Armenia

Received 28 November 2007; received in revised form 8 February 2008
Available online 20 February 2008

Abstract

We have studied the radiation from a relativistic charged particle in a uniform rotation along the equatorial orbit inside a spherical cavity immersed in continuous loss-free dielectric, the permittivity of which in the frequency range under consideration is $\varepsilon > 1$. A formula for calculation of radiation intensity at large distances from the cavity is derived. It is shown that with a special (resonant) choice of a non-dimensional parameter ξ the intensity of synchrotron radiation in the presence of a cavity may be either amplified or reduced almost by $\sqrt{\varepsilon}$ times compared with particle rotation in empty space. The resonance value of ξ is determined by the number of harmonics and is independent of other parameters. A visual explanation of this phenomenon is given and its possible application is discussed.

PACS: 41.60.Ap; 41.75.Ht

Keywords: Synchrotron radiation; Relativistic electron

1. Introduction

Due to unique properties such as the high intensity, high degree of collimation and wide spectral range (see [1–6] and references therein) synchrotron radiation (SR) is used in many disciplines in physics, chemistry, material science and structural biology. These applications show the importance of analyzing various mechanisms for control of SR parameters. The operation of many devices intended for production of electromagnetic radiation is based on interactions of relativistic electrons with matter (see, e.g. [7]). From this point of view it is of interest to study the influence of the medium on spectral and angular distributions of SR.

The characteristics of high-energy electromagnetic processes in the presence of a homogeneous medium are essentially changed, giving rise to new types of phenomena, the well-known example of which is Cherenkov radiation [8–10]. New interesting phenomena occur in the case of inhomogeneous media. A well-known example here is transition radiation. In particular, interfaces between media can be used for monitoring the flow of radiation emitted in various systems. In a series of papers initiated in [11–13] it was shown that the interference between SR and Cherenkov radiation induced at boundaries of spherical or cylindrical configurations leads to new effects. In particular, the investigation of radiation from a charge rotating along an equatorial orbit about/inside a dielectric ball [14–16] showed that when the Cherenkov condition for the ball material and particle speed is satisfied, high narrow peaks in the spectral distribution of the number of quanta emitted to outer space appear at some specific values of the ratio of ball-to-particle orbit radii. In the vicinity of these peaks the rotating particle may generate radiation field quanta exceeding by several dozens of times those generated by the particle rotating in a continuous, infinite and transparent medium having the same real part of permittivity as the ball material. The rise of high power radiation is due to the fact that electromagnetic oscillations of Cherenkov

* Corresponding author. Tel.: +374 10 240577.
E-mail address: levonshg@mail.ru (L.Sh. Grigoryan).

0168-583X/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.nimb.2008.02.013

radiation induced along the trajectory of particle are partially locked inside the ball and superimposed in a non-destructive way. It is worthwhile to point out that a similar phenomenon (less pronounced) takes place in the cylindrical symmetry case. It was shown [17–24] that under a similar Cherenkov condition for permittivity of the cylinder and the speed of particle gyrating about/inside the cylinder, high narrow peaks are present in the spectral-angular distribution for the number of radiated quanta. In the vicinity of these peaks the radiated energy exceeds the corresponding value for homogeneous medium case by several orders of magnitude.

The objective of the present work is to show that the intensity of SR from a relativistic particle in uniform rotation in a cavity immersed in continuous loss-free dielectric may be considerably increased or decreased by proper choice of the radius of cavity and the permittivity of the surrounding substance. The content of paper is organized as follows: in Section 2 the description of the problem and final analytical expressions for the number of quanta emitted from a relativistic particle rotating inside a dielectric cavity are given. Numerical results are presented in Section 3. The cause of radiation amplification is established in Section 4. In the last section the main results of paper are summarized.

2. Formulation of problem and final formula

Consider a relativistic charged particle (e.g. an electron) that rotates in an equatorial orbit inside a spherical cavity (Fig. 1). In properly selected system of spherical coordinates r, θ, φ with the origin in the center of the cavity, the permittivity ε_* of the substance is a step function of radial coordinate

$$\varepsilon_*(r) = 1 + (\varepsilon - 1)\Theta(r - r_1), \tag{1}$$

where ε is the permittivity of the medium surrounding the cavity and r_1 is the radius of the cavity. We shall assume that in the wavelength range under consideration (microwave radiation), the emitted radiation is not absorbed in the medium surrounding the cavity and take the permeability of medium to be equal to unity. The current density may be written as

$$\vec{j}(\vec{r}, t) = \frac{qv\vec{e}_\varphi}{r_q^2}\delta(r - r_q)\delta\left(\theta - \frac{\pi}{2}\right)\delta(\varphi - \omega_q t), \tag{2}$$

where q, $v = r_q\omega_q$ and ω_q are the charge, linear velocity and cyclic frequency of particle rotation and r_q is the radius of particle orbit.

The rotation of the particle entails radiation at some discrete frequencies (harmonics) $\omega_k = k\omega_q$ with $k = 1, 2, 3, \ldots$. It will be assumed that an exterior force (e.g. electric) would make up for the braking of the particle due to the radiation, by forcing the particle to uniformly rotate inside the cavity. At large distances from the sphere the radiation intensity I_k after averaging over the period $T = 2\pi/\omega_q$ of revolution is determined by the expression [25]

$$I_k = \frac{c}{2\pi}\lim_{r\to\infty} r^2 \int \left|\mathrm{rot}\vec{A}_k(\vec{r})\right|^2 \mathrm{d}\Omega, \tag{3}$$

where Ω is the solid angle and $\vec{A}_k(\vec{r})$ is the Fourier component of the vector-potential of the electromagnetic field, that satisfies equation

$$\left(\Delta + \frac{\omega_k^2}{c^2}\varepsilon_*\right)\vec{A}_k(\vec{r}) - \frac{1}{\varepsilon_*}(\vec{\nabla}\varepsilon_*)\mathrm{div}\vec{A}_k(\vec{r}) = -\frac{4\pi}{c}\vec{j}_k(\vec{r}). \tag{4}$$

It is convenient to introduce a non-dimensional quantity

$$TI_k/\hbar\omega_k \equiv n_k, \tag{5}$$

where TI_k is the energy emitted at ω_k frequency during one period of particle revolution and $\hbar\omega_k$ is the energy of the quantum of the corresponding electromagnetic wave. As a result, the total energy W_T emitted for the mean time T, is determined by the expression

$$W_T = \sum_{k=1}^{\infty} n_k\hbar\omega_k. \tag{6}$$

Using the results obtained in [12,15] the characteristics of appropriate SR from the particle have been calculated. In particular, the number of quanta n_k emitted at frequency $\omega_k = k\omega_q$ during one revolution period with cyclic frequency ω_q is determined by the expression

$$\begin{aligned} n_k(\text{cavity};\, v_q, r_1/r_q, \varepsilon) = {} & \frac{8\pi n_q}{\sqrt{\varepsilon}} \sum_{l=k}^{\infty} \frac{1}{l(l+1)} \\ & \times \left[\frac{k}{(2l+1)^2}\left|b_{lk}(E)Y_{lk}\left(\frac{\pi}{2}, 0\right)\right|^2 \right. \\ & \left. + \frac{1}{k}\left|b_{lk}(H)d_{lk}\left(\frac{\pi}{2}\right)\right|^2\right]. \end{aligned} \tag{7}$$

Here $Y_{lk}(\theta, \varphi)$ are the spherical harmonics, $n_q = 2\pi q^2/\hbar c$, $d_{lk}(\theta) = \partial Y_{lk}(\theta, 0)/\partial\theta$ and

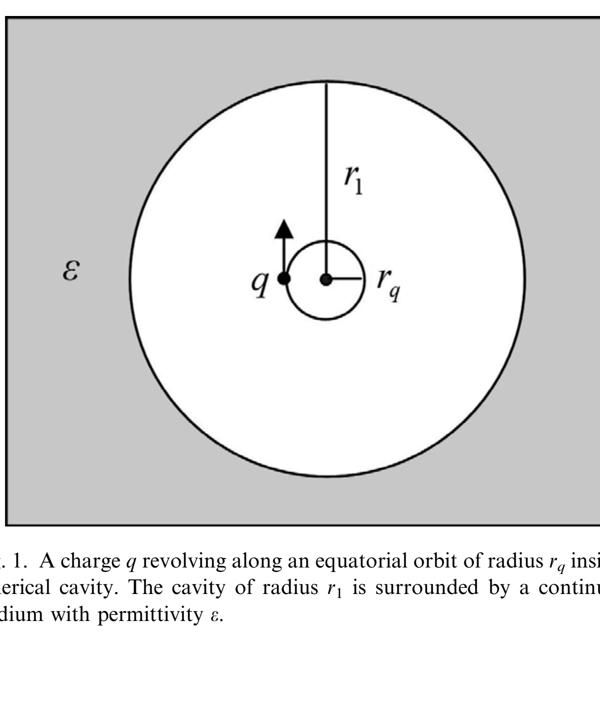

Fig. 1. A charge q revolving along an equatorial orbit of radius r_q inside a spherical cavity. The cavity of radius r_1 is surrounded by a continuous medium with permittivity ε.