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Rarefied gas flow into vacuum through a pipe composed of two circular sections of different radii

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ABSTRACT

The paper is devoted to the study of a rarefied gas flow through a composite circular pipe into vacuum. The pipe is made of two cylindrical sections of different diameters. Two cases are studied: wider pipe followed by narrower pipe (converging configuration) and the narrower pipe followed by the wider (diverging configuration). The analysis is based on the direct numerical solution of the Boltzmann kinetic equation with the S-model collision integral. The results are presented for two length to radius ratios and a large range of Knudsen numbers. The main computed characteristic is the mass flow rate through the pipe. The dependence of the flow field on pipe's geometry and Knudsen number is established. Formation of special features of the flow, such as recirculation zones and a Mach disk, is studied.

1. Introduction

The majority of the computational studies of a rarefied gas flow through a finite-length channels consider straight geometries with the constant cross sectional area [1–3]. Recent examples include the Direct Simulation Monte Carlo (DSMC) studies for short tubes [4–7] and deterministic studies for short, moderate and long pipes [8–13]. A more difficult case is of a flow through curved channel [14,15] or a channel with non-constant cross section [16–18]. In particular, in the latter work [18] a detailed computational analysis of the problem is presented for five pipe's length and several outlet to inlet radii ratios across a wide range of Knudsen numbers.

The present study is devoted to the analysis the rarefied gas flow into vacuum through composite pipes made of two cylindrical sections of different radii. Two possible pipe geometries are considered: the wider pipe followed by the narrower pipe (converging configuration) and the narrower pipe followed by the wider one (diverging configuration). The analysis is based on the direct numerical solution of the Boltzmann kinetic equation with the S-model collision integral [19,20]. The solution of the problem is computed for two pipe lengths and a range of Knudsen numbers. The main goals of the analysis is the calculation of the mass flow rates and investigation of typical flow field features.

2. Formulation of the problem

The formulation of the problem closely follows [6,8–10]. Consider a rarefied gas flow through a circular pipe of length L, connecting two infinitely large reservoirs (volumes) filled with the same monatomic gas. The first half of the pipe, adjacent to the high-pressure reservoir, is of the radius R_1, whereas the second half, adjacent to the vacuum region, is of the radius R_2. The gas in the left reservoir is kept under pressure p_1 and temperature T_1, whereas in the right reservoir the pressure p_2 is so low that it can be regarded as equal to zero. The complete accommodation of momentum and

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energy of molecules occurs at the pipe surface, which is kept under the same constant temperature T_1.

Let us introduce a Cartesian coordinate system (x,y,z) with the origin located in the centre of the inlet section of the pipe $x = y = z = 0$ and the Oz axes directed along the tube. A steady three-dimensional state of the rarefied gas is determined by the velocity distribution function $f(\boldsymbol{x},\xi)$, where $\boldsymbol{x} = (x,y,z)$ are the spatial coordinates, $\xi = (\xi_x,\xi_y,\xi_z)$ is the molecular velocity vector. For the rest of the paper, the non-dimensional formulation is used, in which the spatial coordinates $\boldsymbol{x}$, mean velocity $\boldsymbol{u} = (u_1,u_2,u_3)$, number density n, temperature T, heat flux vector $\boldsymbol{q} = (q_1,q_2,q_3)$, viscosity μ and distribution function f are scaled using the following quantities:

$$R_1,\quad \beta,\quad n_1,\quad T_1,\quad mn_1\beta^3,\quad \mu_1 = \mu(T_1),\quad n_1\beta^{-3}, \tag{1}$$

where $n_1 = p_1/kT_1$ is the number density in the left reservoir, m is the mass of a molecule, $\beta = 2kT_1/m$ is the most probable speed, k is the Boltzmann constant. Below, the non-dimensional variables are denoted by the same letters as the dimensional ones.

The distribution function f is assumed to satisfy the S-model kinetic equation [19,20], which in the non-dimensional variables takes the form

$$\xi_x\frac{\partial f}{\partial x}+\xi_y\frac{\partial f}{\partial y}+\xi_z\frac{\partial f}{\partial z} = \nu\left(f^{(S)}-f\right),\quad \nu = \frac{nT}{\mu}\delta_1,\quad \delta_1 = \frac{R_1p_1}{\mu_1\beta},$$
$$f^{(S)} = f_M\left[1+\frac{4}{5}(1-\Pr)S_\alpha c_\alpha\left(c^2-\frac{5}{2}\right)\right],\quad f_M = \frac{n}{(\pi T)^{3/2}}\exp\left(-c^2\right),$$
$$v_i = \xi_i-u_i,\quad c_i = \frac{v_i}{\sqrt{T}},\quad S_i = \frac{2q_i}{nT^{3/2}},\quad c^2 = c_\alpha c_\alpha. \tag{2}$$

Here δ_1 is the so-called rarefaction parameter, which is inversely proportional to the Knudsen number. Summation over repeated

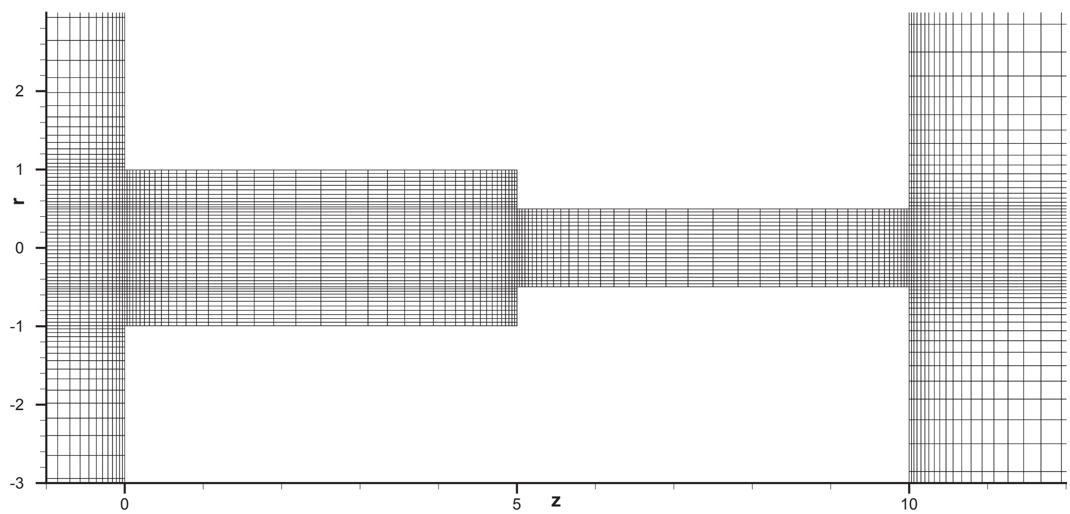

Fig. 1. Symmetry plane cut of the hexahedral mesh for the converging pipe configuration.

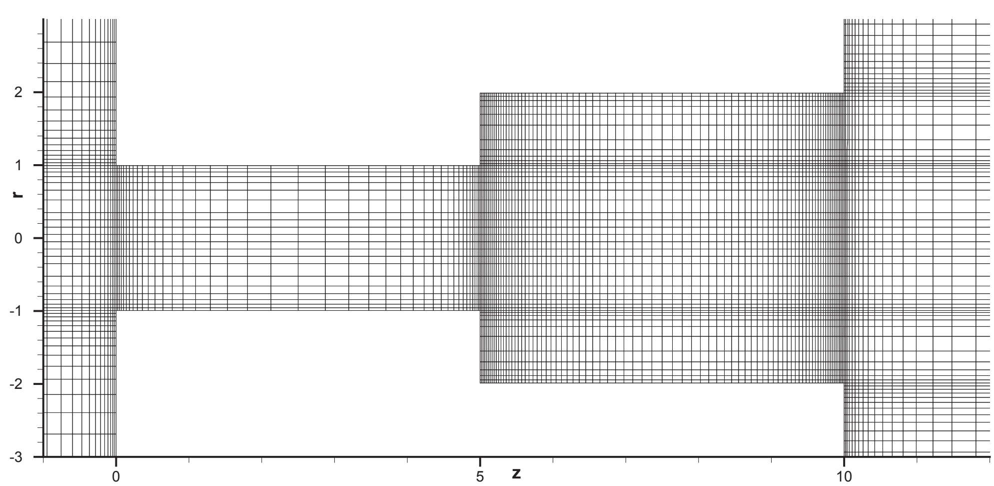

Fig. 2. Symmetry plane cut of the hexahedral mesh for the diverging pipe configuration.