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Pressure driven flows of rarefied gases through long channels with double trapezoidal cross-sections

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ABSTRACT

Steady flows of rarefied single gases through long channels with double trapezoidal cross-section shapes are investigated numerically. The flow is between two large reservoirs having the gas at different pressures. The channels have constant cross-sections along the axial direction. The gas is modeled by the linearized Bhatnagar–Gross–Krook kinetic equation. The diffuse reflection boundary condition is used at the channel walls. The solution of the problem is divided into two stages. At a particular cross-section, the flow is driven by the local pressure gradient. First, the local problem for an arbitrary driving term is solved by using the discrete velocity method. This solution yields the dimensionless flow rate and the velocity profile for a wide range of the gaseous rarefaction. Second, the global flow behavior, i.e. the flow rate and the distribution of the pressure, is deduced for global pressure driven flows on the basis of the conservation of mass. The numerical solution of the kinetic equation is based on the discretization of the spatial and velocity spaces. The spatial space is represented on a rectangular grid. The walls of the channels are aligned parallel to the grid lines or along the diagonal of the grid. Such a choice provides a straightforward calculation of the spatial derivatives. In the interior part of the domain and near the channel walls, second- and first-order finite difference forms are used, respectively. The velocity space is represented by a Gauss–Legendre quadrature. The resulting discrete equations are solved in an iterative manner. The dimensionless flow rates are calculated and tabulated for particular cross-sections in a wide range of the gaseous rarefaction. The flow rate function exhibits the Knudsen minimum. The results are compared to the corresponding ones with other cross-sections. Typical velocity profiles are also shown. Finally, representative results are delivered for global pressure driven flows.

1. Introduction

Flows of rarefied gases through long channels have great practical relevance. These flows can be found in nano- and micro-fluidics [1] or conventional vacuum science [2]. The gaseous rarefaction is characterized by the ratio of the molecular mean free path and the characteristic size of the channel. If the diameter of the capillary is decreased, e.g. in the case of nano- and microflows, or the pressure is reduced, the gas can not be considered continuum and the details of the molecular motions need to be taken into account. The proper description of gaseous flows under rarefied condition should be based on the kinetic level [3]. Such description requires the consideration of the velocity distribution function of the molecules and the Boltzmann or other kinetic equation.

Significant effort has been made to solve kinetic equations for flows in long channel. For such configurations, the flow problem can be divided into two sub-problems. The kinetic equation is typically solved in the two-dimensional cross-section sheet, while the global flow behavior can be obtained on the basis of the conservation of the mass along the axis of the channel. The discrete velocity method has been used to solve the two-dimensional kinetic problem for flows in long capillaries with circular [4,5], rectangular [6], elliptical [7], annular [8], triangular [9,10] or trapezoidal cross-sections [11]. The probabilistic variance-reduced DSMC has also been used to solve some two-dimensional flow configurations [12,13]. If the cross-section has a non-trivial shape, e.g. triangular or trapezoidal, more care is needed to adopt the discrete velocity method. A special spatial grid needs to be used to capture the non-trivial boundary. For triangular and trapezoidal cross-sections, the triangular grid is a sufficient choice. However, the numerical solution of kinetic equations on special grids is still a challenge. Other types of numerical methods, i.e. Ref. [14–16], might also be used to solve kinetic problems. But, these approaches are more complicated and require larger computational effort than the aforementioned ones. For the purpose of the present flow

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configuration, the most suitable choice is to further develop the methods of Refs. [4–11].

In this paper, as a new application, isothermal pressure driven rarefied gaseous flows through long channels with double trapezoidal cross-sections is considered. This flow configuration can be found in microfluidic applications. Special microchannels with this cross-section are manufactured [17,18]. However, numerical results of the flow behaviors in the whole range of the gaseous rarefaction for such channels are not available in the current literature. The gas is modeled by the Bhatnagar–Gross–Krook (BGK) linearized kinetic equation, which is solved by the discrete velocity method. A boundary fitted rectangular grid is used to represent the spatial space. The dimensionless flow rates are calculated and tabulated for various channels and at a wide range of the gaseous rarefaction. Typical velocity profiles are shown. Global pressure driven flows are also considered. The global flow rate and representative pressure profiles are presented for various flow configurations.

2. Statement of the problem

Flows of rarefied gases in long channels with double trapezoidal cross-section shapes are considered. The axis of the channel is along the z' coordinate direction, and its cross-section is in the (x', y') plane. The length of the channel is denoted by L. The layout of the cross-section is shown in Fig. 1. The total width of the channel is $W = 2(a_1 + a_2)$, and its total height is $H = 2b$. The aspect ratio of the channel is defined by H/W, and it is always ≤ 1. The gas is characterized by the local rarefaction parameter

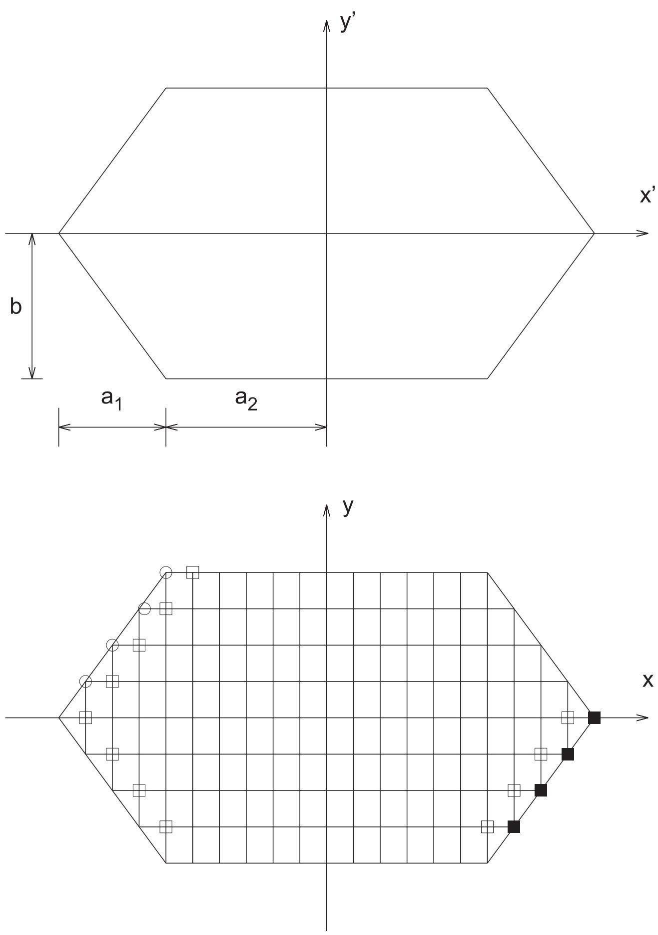

Fig. 1. The hexagonal cross-section (top) and the layout of the spatial grid (bottom).

$$\delta = \frac{PH}{\mu v_0}, \quad (1)$$

where P is the pressure, μ is the viscosity and $v_0 = \sqrt{2k_B T/m}$ is the characteristic speed of the molecules. Here, k_B is the Boltzmann constant, T is the temperature and m is the molecular mass.

The inlet and outlet pressures are denoted by P_1 and P_2. It is assumed that $P_1 > P_2$; hence, the gas flows from the inlet towards the outlet. One of the main interests of this work is in the mass flow rate defined by

$$\dot{M} := \int_{A'} \rho u'_z \mathrm{d}A', \quad (2)$$

where A' denotes the cross-section, ρ is the mass density, u'_z is the axial gaseous velocity.

It is assumed that the channel is sufficiently long, i.e. the length of the channel is much larger than both width and height, $W \ll L$ and $H \ll L$. Under this condition, the flow field depends on the axial coordinate only, the end effects of the channel can be neglected and the local driving force, the dimensionless pressure gradient, is significantly smaller than unity, $X_P = (\mathrm{d}P/\mathrm{d}z')H/P \ll 1$. The gaseous velocity is also negligible compared to the characteristic molecular speed, $u'_z \ll v_0$. This means that the gas is slightly perturbed around equilibrium and a linearized description can be used.

The linearization implies that the local mass flow rate is a linear functions of the local pressure gradient

$$\dot{M} = -\frac{\rho v_0 A'}{2} G X_P, \quad (3)$$

where G is the dimensionless flow rate. This latter quantity has a cardinal importance to determine the dimensional flow rate. G depends only on the local rarefaction of the gas.

The primary scope of the paper is to determine the local velocity profile and G for various values of the rarefaction parameter. Secondly, the global pressure driven flow through the channel is analyzed.

3. Linearized BGK kinetic equation

The problem is modeled at the kinetic level, which is valid in the whole range of the gaseous rarefaction. The flow is described by the linearized version of the Bhatnagar–Gross–Krook kinetic equation. The BGK equation applies a relaxation time approximation for the collision operator. This treatment can provide physically accurate results for isothermal rarefied gas flows; hence, it is suitable in the present case. The linearized BGK equation has been validated against the experimental measurement of the mass flow rate for pressure driven flows [11]. The comparison has yielded good agreement between theory and experiment. The original BGK equation and the derivation of its linearized version are well documented in the literature and can be found in Ref. [19] for example. Interested reader may consult with that work. Here, the formulation of the linearized equation is presented. Since the channel is long, the applicability of the linearized BGK equation is justified as it has been mentioned in Section 2. If the assumption of the long channel is hold, the linearization does not introduce error in the modeling. However, if the channel length is finite, the ratio u'_z/v_0 is higher than in the present case and the full non-linear kinetic equation needs to be solved. It would be interesting to analyze the applicability of the linearized description in terms of the ratios W/L and H/L. However, this analysis is beyond the scope of the present work since it requires a different numerical approach.