Vacuum 109 (2014) 326–332

Contents lists available at ScienceDirect

Vacuum

journal homepage: www.elsevier.com/locate/vacuum

Numerical modelling of thermoacoustic waves in a rarefied gas confined between coaxial cylinders

Denize Kalempa [a,*], Felix Sharipov [b]

[a] Departamento de Ciências Básicas e Ambientais, Escola de Engenharia de Lorena, Universidade de São Paulo, 12602-810 Lorena, Brazil
[b] Departamento de Física, Universidade Federal do Paraná, Caixa Postal 19044, 81531-990 Curitiba, Brazil

ARTICLE INFO

Article history:
Received 17 March 2014
Accepted 11 April 2014
Available online 2 May 2014

Keywords:
Rarefied gas
Thermoacoustic
Sound propagation
Unsteady heat transfer
Pirani sensor

ABSTRACT

Sound waves propagating through a rarefied gas confined between two coaxial cylinders and induced by an unsteady heating of the inner boundary is modelled on the basis of a kinetic model to the linearized and non-stationary Boltzmann equation. The gas flow is considered as fully established so that the dependence of all quantities on time is harmonical. The diffuse scattering of gaseous particles is assumed as the boundary conditions on both cylinders. The solution of the problem depends on three parameters: the Knudsen number, the temperature oscillation frequency and the radius ratio. The deviation of gas properties from their equilibrium values, namely, density, temperature and pressure deviations, and also the bulk velocity and heat flux in the direction of sound propagation are calculated in a wide range of Knudsen number and oscillation frequency in order to cover all the regimes of gas flow, i.e. free molecular, transitional and hydrodynamic regimes. Two values of radius ratio of cylinders are considered to evaluate the curvature effects on the solution of the problem.

1. Introduction

In gaseous systems, an unsteady heating of solid boundary induces sound waves which propagate through the gas and affect its equilibrium properties such as pressure, temperature, etc. These thermally induced waves are usually referred as thermoacoustic waves and its effects are used in many technologies such as heat pumps, refrigerators and mixture separators [1]. Currently, the study of thermoacoustic effects at low pressure is motivated by its application in vacuum and MEMS technologies. For instance, Pirani gauges are widely used for measurements and monitoring pressure in vacuum apparatus. In this kind of device, a heated filament is cooled by the gas surrounding it. Thus, by passing an electrical current through the filament and measuring its temperature, the pressure of the gas can be determined. In spite of its simplicity, advances in microtechnology have demanded the modelling of the dynamical behavior of the gas surrounding the heated filament properly in order to improve the performance of the device. Usually, a modelling of the gas flow in a Pirani gauge is done by considering the steady problem of heat transfer through a rarefied gas confined between coaxial cylinders maintained at different temperatures, see e.g. Refs. [2–8]. However, in practice the temperature of the filament can oscillate with time changing the behavior of the gas in its surround. These effects are not negligible in microscale and can influence the gas pressure significantly.

The aim of the present work is to model the sound propagation through a rarefied gas confined between two coaxial cylinders due to an unsteady heating of the inner cylinder which affects the equilibrium properties of the gas. The propagation of such thermoacoustic waves induces a gas flow in the gap between the cylinders which is characterized by a heat flux and bulk velocity in the direction of sound propagation, and also by deviations of pressure, temperature and density. These macrocharacteristics of the gas flow are calculated for a broad range of gas rarefaction and sound frequency so that the free molecular, transitional and hydrodynamic regimes are covered. A similar problem was already treated in our previous work [9] by considering a gas confined between parallel plates. However, curvature effects play an important role when dealing with gas flows in cylindrical configuration. In order to take into account these curvature effects, another parameter is introduced, namely, the ratio of cylinders radii. The kinetic model proposed by Shakhov [10] to the linearized and non-stationary Boltzmann equation is employed and a discrete velocity method is used to solve it numerically. This kinetic model was already

* Corresponding author.
E-mail addresses: dkalempa@gmail.com, kalempa@usp.br (D. Kalempa), sharipov@fisica.ufpr.br (F. Sharipov).
URL: http://fisica.ufpr.br/sharipov

http://dx.doi.org/10.1016/j.vacuum.2014.04.017
0042-207X/© 2014 Elsevier Ltd. All rights reserved.

successfully employed in our previous works [9,11–14] concerning non-stationary processes in rarefied gases.

In the hydrodynamic regime, the classical equations of fluid dynamics with appropriate temperature jump boundary conditions are solved and the results are compared to those obtained via kinetic equation so that the range of validity of the classical approach for the problem in question is verified.

2. Statement of the problem

It is considered a monoatomic gas confined between two coaxial and fixed cylinders with radii R_0 and R_1 ($R_1 > R_0$). The axis of both cylinders coincide with the z-axis of the Cartesian coordinates. The cylinders are considered too long and, as a consequence, the end effects can be neglected. The temperature of the outer cylinder is constant and equal to the gas equilibrium temperature T_0, while the temperature of the inner cylinder is not constant but it varies on time harmonically as

$$T_{\mathrm{w}}(t) = T_0 + \Delta T_{\mathrm{m}} \Re\left(e^{-i\omega t}\right), \quad \Delta T_{\mathrm{m}} \ll T_0, \tag{1}$$

where ΔT_{m} is the maximum deviation of the temperature from the equilibrium value T_0. $\Re$ means the real part of the complex expression, i is the imaginary unit, t is the time and ω is the temperature oscillation frequency.

The temperature oscillation of the inner cylinder causes the thermo-acoustic waves which propagate through the gas in the radial direction r' and disturb the equilibrium properties of the gas, namely, density n_0 and temperature T_0. Therefore, the disturbed gas is characterized by a density $n(r',t)$ and temperature $T(r',t)$. Moreover, the bulk velocity $U(r',t)$ and heat flux $Q(r',t)$ in the radial direction also appear. We are interested in a pressure tensor denoted by Π in which the diagonal term P_{rr} allows us to determine the deviation of the gas pressure from its equilibrium value p_0, i.e. $P_{\mathrm{rr}} - p_0$, in the direction of waves propagation. In fact, the pressure deviation is the quantity really measured in experiments dealing with propagation of sound waves. However, the heat flux can also be detected experimentally.

The outer cylinder is considered a receptor of sound waves. Its presence can significantly affect the behavior of the gas flow in the gap between the cylinders due to the interference phenomenon between the reflected waves from source and receptor. As a consequence, such an influence must be considered in order to describe the gas flow properly.

The regime of the gas flow induced by the waves propagation is determined by two independent parameters, namely, the rarefaction parameter δ and oscillation parameter θ. These parameters are defined, respectively, as

$$\delta = \frac{R_1 - R_0}{\ell}, \quad \ell = \frac{\mu v_0}{p_0}, \quad v_0 = \left(\frac{2kT_0}{m}\right)^{1/2}, \tag{2}$$

and

$$\theta = \frac{\nu}{\omega}, \quad \nu = \frac{p_0}{\mu}, \tag{3}$$

where μ denotes the viscosity of the gas at equilibrium temperature, m is the molecular mass of the gas and k is the Boltzmann constant. Since the quantity ℓ is the equivalent mean free path of gaseous molecules, the rarefaction parameter is inversely proportional to the Knudsen number. The quantity ν has the order of the intermolecular collision frequency so that the oscillation parameter is the ratio of collision frequency to oscillation frequency. These parameters are independent of each other because one can either change the parameter δ and maintain θ by varying the distance between the cylinders or change the parameter θ and maintain δ by varying the oscillation frequency ω.

Furthermore, since the solution of the problem depends on curvature effects, the parameter $a = R_1/R_0$, corresponding to the ratio between the outer radius and the inner radius of cylinders, is introduced. In the limit $a \to 1$, the solution of the problem tends to that obtained in Ref. [9] for planar geometry.

The dimensionless radial coordinate is introduced as

$$r = \frac{\omega}{v_0} r', \tag{4}$$

where the quantity v_0/ω corresponds to the average distance traveled by gaseous particles during one cycle of the oscillation. As a consequence, the dimensionless radii are written in terms of both rarefaction and oscillation parameters as follows

$$r_0 = \frac{\delta}{\theta}\frac{1}{(a-1)}, \quad r_1 = \frac{\delta}{\theta}\frac{a}{(a-1)}, \quad a = \frac{R_1}{R_0}. \tag{5}$$

Note that the dimensionless distance between the cylinders is given as $L = \delta/\theta$.

The gas flow induced by the oscillatory heating of the inner cylinder is considered as fully established. Consequently, all the quantities describing the gas behavior, i.e. density, temperature, pressure, bulk velocity and heat flux, depends on time harmonically. In order to calculate these quantities, dimensionless complex functions are introduced as

$$\Re\left[\varrho(r)e^{-i\omega t}\right] = \frac{n(r,t) - n_0}{n_0}\frac{T_0}{\Delta T_{\mathrm{m}}}, \tag{6}$$

$$\Re\left[\tau(r)e^{-i\omega t}\right] = \frac{T(r,t) - T_0}{\Delta T_{\mathrm{m}}}, \tag{7}$$

$$\Re\left[\Pi(r)e^{-i\omega t}\right] = \frac{P_{\mathrm{rr}}(r,t) - p_0}{p_0}\frac{T_0}{\Delta T_{\mathrm{m}}}, \tag{8}$$

$$\Re\left[u(r)e^{-i\omega t}\right] = \frac{U(r,t)}{v_0}\frac{T_0}{\Delta T_{\mathrm{m}}}, \tag{9}$$

$$\Re\left[q(r)e^{-i\omega t}\right] = \frac{Q(r,t)}{p_0 v_0}\frac{T_0}{\Delta T_{\mathrm{m}}}. \tag{10}$$

Note that the real part of the complex functions û, τ, Π, u and q represents the deviation of the quantity under consideration from its value in the equilibrium state. Moreover, these complex functions can be represented as

$$\alpha(r) = \alpha_{\mathrm{m}}(r)\exp[i\varphi_\alpha(r)], \quad \alpha = \varrho, \tau, \Pi, u, q, \tag{11}$$

where $\alpha_{\mathrm{m}}(r)$ and $\phi_\alpha(r)$ are real functions corresponding to the amplitude and phase of the complex quantity. Therefore, the calculation of the amplitudes and phases allows us to determine the quantities of physical interest to describe the gas flow under consideration.

3. Kinetic equation

Similarly to our previous works [13,14], the problem is solved on the basis of the kinetic model proposed by Shakhov [10] in its linearized and non-stationary form. As was mentioned in Refs. [13,14], this model is the most appropriate to deal with problems concerning a sound propagation because it provides the