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Conductance calculation of slot channels in laminar gas flow regime at small pressure difference

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ABSTRACT

In vacuum systems gas flows not only through slot channels with constant cross section but also through slot channels with variable cross-section. The relationship obtained analytically for the conductance calculation of three types of slot channels in viscous flow regime at small pressure difference at the ends of the channel is presented in this paper. This relationship is recommended for the calculation of flow rates in vacuum systems channels with variable cross-section with a minimal clearance at a certain point along their length. The experimental study of slot channels conductance was carried out.

1. Introduction

As a rule, formulae for the simplest forms of channels such as pipes, plane slots, etc., are presented [1,2]. But sometimes gas flows through slot channels with intricate shape, with a minimal clearance at a certain point along their length.

This paper is the first one in a series of papers dealing with the conductance calculation of slot channels in different gas flow regimes. This article deals with laminar gas flow in such channels.

2. Calculation of slot channels conductance

The conductance of a channel of any type can be determined with the help of the mass flow rate G

$$U = \frac{GR_G T}{(P_1 - P_2)}, \quad (1)$$

where P_1 and P_2 are pressures at the inlet and the outlet of the channel, respectively; R_G is the individual gas constant; T is the gas temperature.

For viscous flow the mass flow rate can be obtained by numerical solution of gas dynamics equations [3]. But very often a prompt re-calculation of the channel conductance is necessary when geometrical dimensions or gas parameters at the inlet and the outlet of the channel are changed. And for each case a differential equations system should be solved which is not always convenient for applied problems. Moreover, for application of such approach an engineer should have high qualification in the field of numerical solution of gas dynamics equations. One more drawback of such approach is that the conductance calculation of slot channels is often only a small part of mathematical model of complex pumping process which is solved by the successive approximations method. In such a case to solve differential equations system for conductance estimation on every step of calculation is not a simple problem. That is why it is more convenient to use relationships ready for conductance estimation.

That is why the problem of obtaining rather simple relationships giving the acceptable calculation accuracy is urgent. The attempt to obtain such relationships was made earlier [4,5] where for the calculation of the mass flow rate in laminar flow regime a relationship for a long plane rectangular channel with the constant cross-section [1,2] was used

$$G = \frac{L\delta^3}{12 R_G T \eta} P \frac{dP}{dx}\left[1 - \frac{192}{\pi^5}\frac{\delta}{L}\sum_{j=0}^{\infty}\frac{1}{(2j+1)^5}\cdot \tan\left(\frac{(2j+1)}{2}\frac{\pi L}{\delta}\right)\right], \quad (2)$$

where δ is the minimal clearance between the channel walls; L is the channel width (the dimension in the direction perpendicular to the sheet of paper); x is the coordinate along the channel; η is the gas dynamic viscosity coefficient.

This relationship can be used only for the cases when the average pressure $P_{av} = (P_1 + P_2)/2$ in a channel is considerably higher than the pressure difference at the ends of the channel, that is pressure ratio $\tau = P_2/P_1 \to 1$ [6]. According to this relationship the

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conductance is a linear function of the average pressure. But in reality even a slight deviation of τ from one results in a deviation from linearity of the relationship $U = f(P_{av})$.

For the conductance calculation of slot channels formed by curvilinear walls a first order differential equation was obtained [4,5] which gives the changing of pressure along the slot length. But the necessity of numerical solution of this equation causes again considerable inconvenience when it is used for applied problems.

For conductance calculation of channels with variable cross-section an approach based on numerical solution of Boltzmann kinetic equation exists.

In a number of articles [7–10] authors consider circular pipes with variable radius along their length. In [7] they use a rarefaction parameter $\sigma = \sqrt{\pi}/(2Kn) = P\delta/(\eta\upsilon)$, where υ is the gas velocity. The data are given there for $\sigma = 0{,}01\ldots10$, which corresponds to molecular and transition gas flow. In rare cases σ is as high as 100, which corresponds to the boundary between viscous and transition gas flow. The authors of these articles suggest that this approach is used for conductance calculation in laminar gas flow regime. But the necessity of numerical solution of these equations when the initial parameters vary over and over again causes above mentioned inconvenience in application of this approach for applied problems as well as in application of [3].

In our work we consider viscous gas flow, and a simple formula which makes it possible to calculate rather promptly the conductance of channels formed by cylindrical walls (Fig. 1) is presented in this work.

To obtain relationships for the conductance of the channels 1–3 in laminar flow regime Formula (2) is used by analogy with [5,6] in the following form

$$U = \frac{\delta^3 L}{12\eta}\cdot\frac{P_1+P_2}{2l}\left[1-\frac{192\delta}{\pi^5 L}\left(\tanh\frac{\pi L}{2\delta}+\frac{1}{3^5}\tanh\frac{3\pi L}{2\delta}+\ldots\right)\right]. \quad (3)$$

where l is the length of a channel in the direction of gas flow.

For the slot channels under study $L >> \delta$, which gives the opportunity to reduce (3) to the form (4) for the specific conductance

$$U_{sp} = \frac{U}{L} = \frac{\delta^3}{12\eta}\cdot\frac{P_1+P_2}{2l}. \quad (4)$$

Let's divide the channel under study (Fig. 2) into n rectangular elementary channels within which the clearance can be considered constant $\delta_i = \text{const}$. We shall consider that the assumptions taken for each elementary channel are the same as those taken for the whole channel (gas flow is steady, local resistance at the inlet is absent).

Then we shall use the specific resistance of the channel W_{sp}. For each elementary channel this value can be written as

$$W_{sp_i} = \frac{1}{U_{sp_i}} = \frac{12\eta}{\delta_i^3}\frac{\Delta l}{P_{av_i}}. \quad (5)$$

The resistance of the whole channel can be determined as the sum of resistances of n elementary channels connected in a

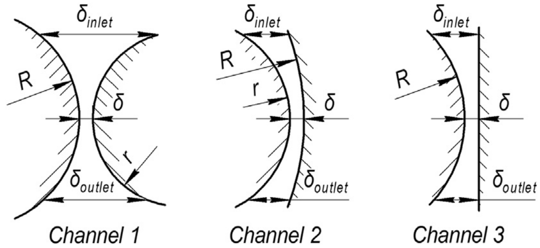

Fig. 1. Types of channels under study.

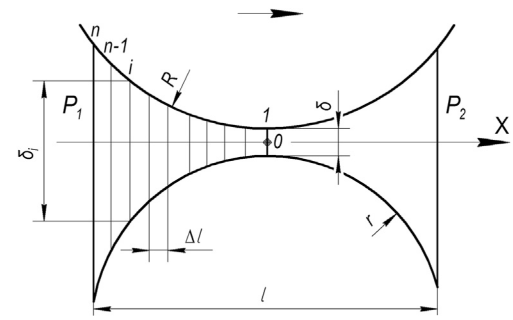

Fig. 2. To the conductance calculation of a variable cross-section channel.

series$W_{sp} = \Sigma_{i=1}^{n} W_{sp_i}$. Denoting the abscissas of the inlet and outlet cross-sections by $-l/2$ and $l/2$ we obtain

$$W_{sp} = \int_{-l/2}^{l/2} \frac{12\eta}{\delta(x)^3}\cdot\frac{1}{P_{av}(x)}dx. \quad (6)$$

If pressure difference between the inlet and the outlet is small we can write

$$W_{sp} = 12\eta\frac{1}{P_{av}}\int_{-l/2}^{l/2}\left(\frac{1}{\delta(x)^3}\right)dx. \quad (7)$$

This equation makes it possible to calculate the resistance of slot channels of any shape. Its solution is determined by a form of the function $\delta(x)$ (the clearance changing along the channel length). For the channels 1–3 the function $\delta(x)$ is given as

$$\delta(x) = \delta \pm R + r \mp \sqrt{R^2 - x^2} - \sqrt{r^2 - x^2}. \quad (8)$$

When the relationship (8) is substituted into (7) the Equation (7) has no analytical solution.

If we substitute the channel wall profiles by parabola arcs $y = a_1x^2$, $y = a_2x^2 + \delta$ (Fig. 3) the Equation (7) can be analytically solved in the form

$$W = \frac{3\eta\left(3\arctan\sqrt{\alpha-1}+\sqrt{1/\alpha-1/\alpha^2}\cdot(2/\alpha+3)\right)}{P_{av}(a_2-a_1)^{1/2}\delta^{5/2}L}, \quad (9)$$

where $\alpha = \delta_{inlet}/\delta$.

The deviation of parabolas from circumferences at peripheral sections of the channel may be rather large but as it will be shown it hardly influences the total resistance of the channel. The relationship between the ratio of the section resistance W to the whole channel resistance W_∞ and α plotted on the basis of the relationship (9) analysis is presented (Fig. 4). Fig. 4 shows that the channel

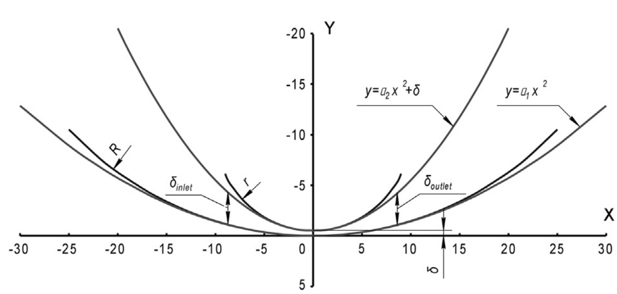

Fig. 3. Parabolic circumference approximation.