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Air damping models for micro- and nano-mechanical beam resonators in molecular-flow regime

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ABSTRACT

Researches on energy dissipation of micro- and nano-mechanical beam resonators into the surrounding air are classified into two models, air drag force damping of the resonant beam vibrating in free space and squeeze film air damping of the resonant beam vibrating nearby walls. Appropriate application of these two models are crucial to the design of high-Q micro- and nano-mechanical beam resonators. In this paper, we will present a detail analysis on appropriate application range of these two models for evaluating air damping in micro- and nano-mechanical beam resonators. The two models are applied for investigating damping in the cantilevered and bridged beam resonators. The obtained analysis results show the significance difference between the two models for estimating the quality factor of the micro- and nano-mechanical beam resonators. The criterion for the distinction between the two models is derived by using the ratio factor of damping coefficients of micro- and nano-mechanical beam resonators.

1. Introduction

Micro-and nano-mechanical beam resonators are promising for applications in timing, sensing, and physical measurements [1–4]. The requirement of a resonator for such applications is high quality factor Q. The resonator with high Q supplies high sensitivity, low noise, and low energy consumption. Besides the demand on high Q, resonators are also required to operate at high frequency, for example, to enhance the time resolution of measurement in atomic force microscopy and real-time measurements [5]. In order to increase the resonant frequency, the size of the resonator needs to be decreased. However, this leads to high surface to volume ratio, the air damping is impossible to neglect in micro- and nano-mechanical beam resonators. The quality factor thus decreases when the size of resonators decreases due to increasing damping effect. Micro- and nano-mechanical beam resonators can be employed in vaccum environment or in air environment [4,6–8]. When packaged in high vacuum, the air damping in micro- and nano-mechanical resonators is eliminated [9,10]. However, in many applications, the device is required to operate in high damping conditions such as in air and liquid environments [2]. The energy dissipation caused by high damping environments thus needs to be considered in initial design process. Different models for evaluating air damping mechanisms in different vibration structures from viscous flow regime to molecular flow mechanism have been introduced in literature [9–17]. A model for evaluating precisely the air damping of a resonator in molecular flow mechanism is important for prodicting ultimate Q value that a resonator can achieve. It generates from the fact that the package is not easy to obtain adequate high vacuum to eliminate completely air damping in the resonator. There are few approaches for analyzing the air damping in molecular flow mechanism introduced in literature which can be classified into two models. The first approached model is for estimating the air damping of resonators with the resonant beam vibrating in free space [9–11,15,16]. In Refs. [11], the author presented a model for investigating the damping of the oscillating vane resonator based on considering the pressure difference between the front and back sides of the vane. The pressure difference is determined by considering the change rate of momentum of a molecule striking the surface of the vane using the Maxwell-Boltzman velocity distribution function for gases. However, using this model for estimating the damping of a resonator oscillating nearby walls, the quality factor is overestimated [12,18,19]. In order to bring the results from the theoretical model close to the experimatal data, later efforts have been done by modifying Christian's theoretical model [13,16]. In Refs. [18], M. Bao pointed out that the modifications are not correct and the

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expressions used in evaluating the quality factor by previous authors have not considered the effect of nearby walls. This leads to the second model which considers damping effect caused by squeeze film air in the actuation gap [18,20]. Recently, the transition between these two models by verifying the gap between the oscillating object and nearby walls has been investigated both theory and experiment [21]. Understanding the effect of each model is very important for their application in predicting adequately the quality factor and operation characteristics of micro- and nano-mechanical beam resonators.

In this study we present models for evaluating quantitatively the air damping effect of micro- and nano-mechanical beam resonators in cantilevered and bridged configurations. We consider two damping effects relating to resonant beams vibrating very far away from and very close to walls. The difference between these two damping models is analyzed in detail by explicitly theoretical expressions. We will examine how this difference varies with the geometry of the resonator. Finally, a general rule for determining the application range of the two models is derived.

2. Models for damping analysis in molecular flow mechanism

Different damping mechanisms are distinguished as the ambient air pressure varies. Based on the Knudsen number K_n, the air flow is divided into four regimes, namely, continuum flow when $K_n < 0.01$, slip flow when $0.01 < K_n < 0.1$, transitional flow when $0.1 < K_n < 10$ and free molecular flow when $K_n > 10$. K_n is the ratio between the mean-free path of the gas λ and the characteristic dimension l_c of the resonator. The mean-free path is calculated by $\lambda = k_B T/\sqrt{2}\pi P d^2$. Here d is the diameter of the gas molecule and P is the ambient air pressure. l_c can be the width w of the resonant beam or the gap g between the vibration beam and nearby walls, which depends on the beam vibrating in free space or close to walls. The geometry of resonators is used for investigating air damping shown in Fig. 1. We illustrate pressure P_m, where the different flow regimes occur, as a function of l_c in Fig. 2. In this case, we assume that the environment is air at room temperature. When the dimension of resonator is reduced, K_n increases. So, the molecular flow condition is satisfied at a higher pressure value. In general, K_n is inversely proportional to P and l_c. If l_c increases with a scale factor, P_m has to be decreased with a corresponding scale factor in order to keep the molecular flow condition satisfied, i.e. the damping models in the molecular flow region are valid. In the molecular flow region, the interaction between gas molecules is neglected and the damping is caused by independent collisions of non-interacting air molecules with the surface of vibrating beam.

The Knudsen number is a dimensionless parameter used to classify flow mechanisms based on the geometry parameters of resonator. Another dimensionless parameter is also necessary to define the change of flow being the Weissenberg number W_i, which

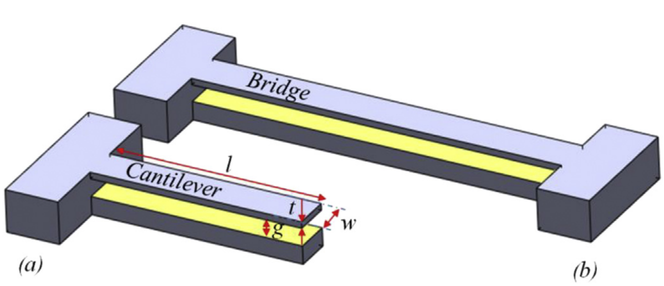

Fig. 1. Schematic of the resonators used for air damping analysis with the resonant beam clamped at one end (cantilevered beam configuration) (a) and the resonant beam clamped at both ends (bridged beam configuration) (b).

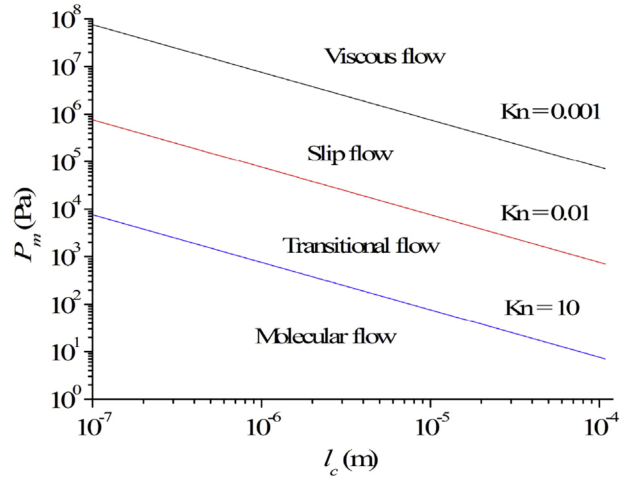

Fig. 2. Pressure P_m, where the molecular flow condition starts to occur, is illustrated as a function of the characteristic dimension l_c of the resonator. l_c can be the width w of resonant beam in the case of the beam vibrating in free space or the gap g between the vibration beam and nearby wall in squeeze film air damping.

associates with the oscillation frequency ω of the resonator and the fluidic relaxation time τ in the medium as $W_i = \omega\tau$ [22]. Using W_i, one can define the limits of validity of Newtonian hydrodynamics, where the continuous flow theory can be employed for calculating damping in the resonator. If $W_i << 1$, the fluidic flow satisfies the assumptions of Newtonian hydrodynamics. When $W_i \approx 1$, the fluidic flow is in the transition mechanism. For $W_i >> 1$, the assumption of Newtonian hydrodynamics is no longer valid, so the free molecular flow theory needs to be taken into account. In studying the interaction of resonators with a gaseous medium, τ is found out being proportional to $1/P$ [22]. So, in order to vary W_i, we can change either the resonance frequency or operation medium pressure of the resonator. Thus, damping model for the resonator in each flow mechanism depends on the interplay of the two dimensionless parameters, K_n and W_i, respectively, the geometry parameters, the resonance frequency, and operation medium pressure of the resonator.

In the study of air damping in micro- and nano-mechanical beam resonators, two models have been introduced [9–11,15,16,18,20,21]. The first model is for beam structures vibrating in free space, i.e. without affected by nearby walls [9–11,15,16]. In this case, the most common used model is Christian's model [11]. When compared with experimental data by Zook et al. [19] and Guckel et al. [23], the quality factor is overestimated by about an order of magnitude. For example, at 1 Torr (133 Pa) the Q value is calculated by Christian's model as $Q = t\rho_b\omega\sqrt{\pi/32}\sqrt{RT/M_g}/P$ being 1.645×10^3 while the experimentally measured value is about 2×10^2 [19]. Here, t (= 1.8 μm) is the thickness of resonant beam ρ_b(=2.33 g/cm^3) is the density of the polysilicon beam material, ω(=566 kHz at T = 30 °C) is the natural resonant frequency of the beam, M_g (= 28.97 g/mol for dry air) is the molar mass of the gas, and R (= 8.31 kg m^2/(s^{2}K)) is the universal molar gas constant. In order to reduce the difference between the theoretical results and the experimental data, Kadar et al. [13] modified the calculation for Christian's model by applying the Maxwell-stream distribution describing the velocity distribution of the molecules striking a surface instead of in a gas as a whole. Then, Li et al. [16] modified Maxwell-stream velocity distribution function used by Kadar et al. However, Kadar et al.'s modification is considered to be incorrect [12,18]. The discrepancy between the results by Christian's theoretical model and the