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Disk spin-down measurements in the free-molecular flow regime: A new method for measurement of tangential momentum accommodation coefficients

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ABSTRACT

An experimental technique using spin-down of a disk approaching the free-molecular flow limit is developed to measure the tangential momentum accommodation coefficient σ_t. The new technique uses a disk mounted on a shaft supported by gas bearings in vacuum. A differential scavenging system allows disk spin-down experiments in different gases. Representative test results with two different materials in two different gases are shown, demonstrating the method is capable of handling a range of gases, as well as any material that is available as, or can be formed into, a planar disk.

1. Introduction

In highly rarefied flows, momentum transfer between gas molecules and surfaces is quantified by the tangential momentum accommodation coefficient, σ_t. This variable is a measure of the amount of momentum that is transferred from a gas molecule to a surface. When σ_t is 0, a gas molecule strikes a surface and bounces back without transferring any momentum to the surface. Such an interaction between a gas molecule and a material surface is known as 'specular' reflection. In the opposite case of σ_t equal to 1, a gas molecule is absorbed by the material surface and is then re-emitted in random directions with the velocity being set by the wall temperature. This kind of interaction is known as 'diffuse' reflection. Fig. 1 shows the two modes of interaction between gas molecules and a surface [1]. Previous experimental measurements have shown that σ_t generally varies between 0 and 1 [2].

σ_t is the ratio of the difference of incident and reflected momentum fluxes to the incident momentum flux. This is defined by Eq. (1) [1].

$$\sigma_t = \frac{\Phi_i^{(mv_t)} - \Phi_r^{(mv_t)}}{\Phi_i^{(mv_t)}}, \tag{1}$$

where $\Phi_i^{(mv_t)}$ is the incident momentum flux and $\Phi_r^{(mv_t)}$ is the reflected momentum flux. The incident momentum flux is obtained from the Maxwell–Boltzmann distribution function for a gas molecule of mass m and traveling with an incident velocity v_t. For a surface meeting a flow with a velocity u at an incident angle α, the incident momentum flux is given by Eq. (2) [1].

$$\Phi_i^{(mv_t)} = P_g\left\{s_t\frac{\exp(-s_3^2)}{\sqrt{\pi}} + s_t s_3[1 + erf(s_3)]\right\}, \tag{2}$$

where P_g is the ambient gas pressure. The dimensionless velocities s_3 and s_t are defined by Eqs. (3) and (4) [1]:

$$s_3 = \sin(\alpha)u\sqrt{\frac{m}{2kT_g}} = u_3\sqrt{\frac{m}{2kT_g}}, \tag{3}$$

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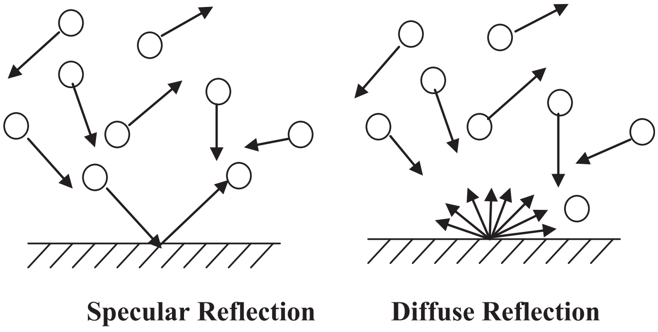

Fig. 1. Comparison Of specular and diffuse reflection.

$$s_t = \cos(\alpha) u \sqrt{\frac{m}{2kT_g}} = u_t \sqrt{\frac{m}{2kT_g}} \tag{4}$$

where m is the mass of the gas molecule (48.1×10^{-27} kg for air), k is the Boltzmann constant, T_g is the gas temperature, u_3 is the random particle normal velocity, u_t is the particle tangential velocity, and α is the incident angle.

In the free-molecular flow regime, the normal pressure P and shear stress τ acting on a surface element will be given by Eqs. (5) and (6) [1]:

$$P = P_g \left[\left(\frac{2-\sigma_n}{\sqrt{\pi}} s_3 + \frac{\sigma_n}{2} \right) \left(\exp\left(-s_3^2\right) + s_3 (1 + erf(s_3)) \sqrt{\pi} \right) + \left(\frac{2-\sigma_n}{2} (1 + erf(s_3)) \right) \right], \tag{5}$$

$$\tau = \sigma_t P_g s_t \left[\frac{\exp(-s_3^2)}{\sqrt{\pi}} + s_3 [1 + erf(s_3)] \right], \tag{6}$$

where σ_t is the tangential momentum accommodation coefficient for a specific gas versus material surface interaction. σ_n is the normal momentum accommodation coefficient.

The values of the momentum accommodation coefficients are of interest to a range of flows of engineering interest, including aerospace applications such as planetary entry [3–5]. Recent work [6] shows that prediction of drag in aero-capture, where the spacecraft skims the upper atmosphere as part of entering a planet's orbit [7], is extremely sensitive to momentum accommodation coefficients.

Flow in multiple micro- and nano-scale geometries is influenced by the surface effects captured by the momentum accommodation coefficient. Micro-channel flows, where the flow can go from continuum to free-molecular due to high pressure drops, will be affected by slip and free-molecular boundary conditions [8]. The effects are seen much more dramatically in devices such as hard-disk drives [9] and micro- and nano-scale resonators [10]. In many cases, free molecular losses are dominant. Accurate prediction of these losses depends on using linearized versions of Eqs. (5) and (6) that are very sensitive to changes in accommodation coefficients [11]. Finally, at the extreme scale of porous media, continuum break-down effects are often noticeable [12,13] – an effect that has recently attracted interest in modeling of shale-gas reservoirs [14,15].

The potential for momentum accommodation to affect micro- and nano-scale flows has been known for over 100 years, leading to multiple attempts to measure these values. One of the earliest attempts was by Millikan and his co-workers, who performed a co-axial cylinder experiment to measure momentum accommodation [16] to determine the accuracy of their famous oil-drop experiment [17]. They rotated the outer cylinder and the torque on the surface of the inner cylinder was measured, calculating σ_t from the measured torque. Chambers et al. measured the torque on a stationary disk by placing it near a rotating disk [18]. Gabis and his co-workers magnetically levitated a spinning metal sphere and measured the deceleration torque with the sphere spin down [19] in the slip-flow regime, while other researchers performed similar measurements in the free-molecular flow regime [20]. Gronych et al. measured σ_t using a viscosity vacuum gauge with a vibrating metal ribbon [21]. Bentz et al. performed experiments with the spinning rotor gauge in the continuum and slip flow regimes with nitrogen and methane [22]. Other recent work has used measuring the pressure drop of gas flow through a micro-channel to determine the accommodation coefficient [23], requiring the material to be micro-machine-able. The majority of these measurements have been done in the continuum slip flow regime and the transitional flow regime. For micro-channel measurements, the extreme pressure drop may result in the flow going from continuum to free-molecular within the same system. In the present work, a new experimental technique is reported that is capable of σ_t measurements in the free molecular flow regime.

2. Knudsen number

As the continuum assumption breaks down, the value of momentum accommodation will increase in significance [1]. This will be determined by the Knudsen number Kn:

$$Kn = \frac{\lambda}{L}, \tag{7}$$

where λ is the mean free path between collisions, and L is the length scale. The mean free path can be found from kinetic theory and is given by Eq. (8):

$$\lambda = \frac{1}{\sqrt{2} P_g \pi} \frac{kT_g}{d_m^2}, \tag{8}$$

where d_m is the collision diameter of the gas, equal to 4.19×10^{-10} m for air.

For an object with a length of approximately 0.005 m under atmospheric conditions, the Knudsen number based on length is of the order of 10^{-5}. The degree of rarefaction increases with increasing Knudsen numbers: for a Knudsen number of 0.005–0.1, the Navier–Stokes equations with slip and temperature boundary conditions are generally accurate. As the Knudsen number increases above 0.1, the assumptions of the Navier Stokes equations breakdown, leading to transitional flow. Kn increases with increase in the degree of rarefaction. At a Knudsen number between 1 and 10, depending on the geometry, the flow becomes free-molecular [24].

Previous experiments showed that for a rotating disk, the breakdown in continuum in associated boundary layer flows can be observed as a viscous scaling law used for non-dimensionalizing disk deceleration torque in the viscous flow regime fails. For aluminum disks of diameters ranging from 0.15 m (6 inches) to 0.21 m (8.25 inches), and thickness of 0.5 mm rotating in air, the free molecular flow regime is encountered at gas pressures of 1 Pa or less, where Kn is 1 or larger [25]. The following section shows some details from this work.