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Short communication

Dependence of loss rate of electrons due to elastic gas scattering on the shape of the vacuum chamber

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ABSTRACT

The loss rate of electrons of stored electron beam due to elastic scattering with the nuclei of residual gas atoms in an electron storage ring depends upon the shape factor which is governed by the shape and size of vacuum chamber. As the vacuum pressure along the circumference of storage ring is normally not uniform so the shape factor as a function of longitudinal position is required to be known. In this paper, expressions for the shape factor for a rectangular and an elliptical vacuum chamber as a function of longitudinal position along the circumference in a storage ring are derived and applied for the estimation of shape factor in Indus-2 storage ring. Using measured vacuum pressure and shape factor along the longitudinal position, the beam loss rate in Indus-2 due to elastic gas scattering with residual gas atoms considering rectangular and elliptical shape of vacuum chamber are also reported.

For the storage of an electron beam for a long lifetime in an electron storage ring, average vacuum pressure ~1×10^{-9} mbar is required in the vacuum chambers in which the electron beam circulates. The presence of residual gas species within a vacuum chamber causes scattering of the electrons of the stored beam with the nuclei of residual gas atoms. The scattering may be elastic or inelastic [1–3]. In this paper we discuss the loss of electrons in a storage ring due to elastic coulomb scattering of electrons with the nuclei of residual gas atoms present in a vacuum chamber. The loss of electrons due to this scattering takes place at minimum acceptance location i.e. at maximum β function in horizontal (X) and vertical (Z) planes, when the aperture of vacuum chamber is uniform in the ring and depends on the parameter known as shape factor, which is governed by the shape and size of the vacuum chamber. Rectangular or elliptical shapes of the vacuum chamber are widely used to store electrons in electron storage rings. The vacuum pressure in a storage ring normally varies from place to place and sometimes the vacuum pressure in one part of the ring is very poor compared to the other parts. Under these conditions, in order to estimate loss rate of electrons due to elastic scattering, we have to use the vacuum pressure and shape factor information along the longitudinal position in the ring. It is thus important to know the vacuum pressure and shape factor as a function of the longitudinal position in the ring. The average shape factor for a rectangular and elliptical chamber is discussed in Ref. [4] and used for loss rate estimation in electron storage rings like MAX II [5], SPEAR3 [6], INDUS-2 [7], SAGA-LS [8] etc.

We derive expressions for the shape factor as a function of longitudinal position along the circumference of a storage ring for rectangular and elliptical shape of vacuum chamber starting from ab-initio using linear beam dynamics. The motion of electrons is considered to be constrained by the physical aperture neglecting the nonlinear beam dynamical effects. Here, the position of electrons at the focusing quadrupole is transformed to the defocusing quadrupole location based on the β functions in horizontal and vertical planes and vice versa to define the part of the vacuum chamber at which the beam loss takes place at these locations. This approach has enabled derivation of exact expressions for the shape factor as a function of the longitudinal position within the domain of linear beam dynamics.

The loss rate of relativistic electrons of stored beam due to elastic coulomb scattering with the nuclei of residual gas atoms $1/\tau_{elastic}$ [9] is given by

$$\frac{1}{\tau_{elastic}} = \left\langle -\frac{1}{N}\frac{dN}{dt}\right\rangle = c\langle n\sigma\rangle \quad (1)$$

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where N, c, n, σ are the number of electrons, speed of electrons, residual gas density and scattering cross section of electrons respectively, $\langle\rangle$ denote the average over the ring and $\tau_{elastic}$ is the beam lifetime due to elastic scattering.

When an electron of stored electron beam at longitudinal position s_0 collides elastically with the nucleus of a residual gas atom in a storage ring, it gets deflected by an angle θ. In spherical polar coordinates, for small scattering angle, the deflection is resolved as $\theta_x = \theta cos\phi$ in X plane and $\theta_z = \theta sin\phi$ in Z plane, where ϕ is azimuth angle. For minimum scattering angle $\theta = \theta_m$ at scattering location, the electrons are lost at maximum β_x or at maximum β_z locations in the storage ring.

If the electron scattered at longitudinal position s_0 reaches the boundary of the vacuum chamber at position s_1, its horizontal x and vertical z coordinates at s_1 are given as

$$x(s_1) = \sqrt{\beta_x(s_0)\beta_x(s_1)}\theta_x = \sqrt{\beta_x(s_0)\beta_x(s_1)}\theta_m \cos\phi \quad (2)$$

$$z(s_1) = \sqrt{\beta_z(s_0)\beta_z(s_1)}\theta_z = \sqrt{\beta_z(s_0)\beta_z(s_1)}\theta_m \sin\phi \quad (3)$$

where $\beta_x(s_0)$ *and* $\beta_z(s_0)$ are β functions at the scattering location s_0 in X and Z planes respectively and $\beta_x(s_1)$ *and* $\beta_z(s_1)$ are β functions at beam loss location s_1 in X and Z planes respectively and θ_m is the minimum scattering angle for electron loss. In equations (2) and (3) as well as everywhere in this paper, maximum betatron displacements are considered taking the betatron phase term equal to one because here only those electrons, which are lost from the ring, are taken into account.

The elastic scattering cross section, causing loss of electrons scattered at the location j in a storage ring is given as [9–11].

$$\sigma_j = \frac{2Z^2r_0^2}{\gamma^2}F_j \quad \textit{where } F_j = \int_0^{2\pi}\frac{d\phi}{\theta_m^2(\phi)} \quad \text{is defined as the shape factor} \quad (4)$$

where Z: atomic number of residual gas atom, r_0: classical electron radius and γ: relativistic factor. Using residual gas density $n = P/kT$, where P: residual gas pressure, k: Boltzmann constant and T: temperature, equation (1) becomes

$$\frac{1}{\tau_{elastic}} = \frac{c}{kT}\left\langle\sigma\cdot P\right\rangle \quad \text{where } \langle\sigma\cdot P\rangle = \frac{2Z^2r_0^2}{\gamma^2}\langle F\cdot P\rangle \text{ and } \langle F\cdot P\rangle = \frac{\sum_{j=1}^{l}F_jP_j}{l} \quad (5)$$

Here $\langle F\cdot P\rangle$ is the average of product of shape factor and vacuum pressure at each scattering location and l is the number of scattering locations spread uniformly over the ring. The beam loss rate $1/\tau_{elastic}$ is proportional to $\langle F\cdot P\rangle$. Obviously, the correct value of $\langle F\cdot P\rangle$ is essential to estimate the beam loss rate.

Substituting $1/\theta_m^2$ from equations (2) and (3) into equation (4), the shape factor F at scattering location j becomes

$$F_j = \int_0^{2\pi}\frac{\beta_x(j)\beta_x(s_1)\cos^2\phi + \beta_z(j)\beta_z(s_1)\sin^2\phi}{x^2(s_1)+z^2(s_1)}d\phi \quad \textit{where } \tan\phi = \sqrt{\frac{\beta_x(j)\beta_x(s_1)}{\beta_z(j)\beta_z(s_1)}}\frac{z(s_1)}{x(s_1)} \quad (6)$$

Dependence of F on x and z indicates that it is governed by the shape and size of the vacuum chamber. We consider here that the vacuum chamber has uniform cross section all along the circumference. In such a ring, the beam loss will takes place either at the location where β_x is maximum or where β_z is maximum.

To derive the expression of shape factor for rectangular vacuum chamber, let the horizontal and vertical apertures of the chamber be $\pm a$ and $\pm b$ respectively. In order to find out the domain of azimuth angle ϕ in equation (6) for electron loss in X and Z plane, we consider that at maximum β_z location, the electrons lie on the boundary of vacuum chamber i.e. the electron, which is at $P(a,b)$ at maximum β_z location, will be at $P''(\sqrt{\beta_{xm}/\beta_x}a, \sqrt{\beta_z/\beta_{zm}}b)$ at maximum β_x location as shown in Fig. 1(a), where β_{xm} and β_z are β functions at maximum β_x location in X and Z plane respectively and β_x and β_{zm} are β functions at maximum β_z location. The electrons which lie on the boundary of the chamber at maximum β_z will, accordingly, follow the dotted rectangle at maximum β_x location as shown in Fig. 1(a). Here the solid rectangle shows the physical aperture at maximum β_x location. Similarly, at maximum β_x location, electrons are on the boundary of the vacuum chamber i.e. the electron, which is at $P(a,b)$ at maximum β_x location will be at $P''(\sqrt{\beta_x/\beta_{xm}}a, \sqrt{\beta_{zm}/\beta_z}b)$ at maximum β_z location as shown in Fig. 1(b). The electrons which lie on the boundary of the chamber at maximum $\beta_x$will follow the dotted rectangle at maximum β_z location as shown in Fig. 1(b) whereas the solid rectangle shows the physical aperture at maximum β_z location.

From Fig. 1(a), it is seen that at the location of maximum β_x, electrons, which have the magnitude of horizontal displacement a or greater and also vertical displacement magnitude up to $OB'(\sqrt{\beta_z/\beta_{zm}}b)$ are lost on the $P'AS'$ and $Q'A_1R'$ parts of the vacuum chamber. Electrons, having the magnitude of vertical displacement greater than to OB' are lost at the maximum β_z location.

Similarly, from Fig. 1(b), it is understood that at the location of maximum β_z, electrons, which have the magnitude of vertical displacement b or greater and also magnitude of horizontal displacement up to $OA'(\sqrt{\beta_x/\beta_{xm}}a)$ are lost on the$P'BQ'$ and $S'B_1R'$ parts of the vacuum chamber. Electrons having the magnitude of horizontal displacement greater than to OA' are lost at the maximum β_x location.

From Fig. 1(a), the coordinate of the electron at the location P' is $(a, \sqrt{\beta_z/\beta_{zm}}b)$ where $\beta_{zm} > \beta_z$. Using equations (2) and (3), the azimuth angle ϕ is related to the coordinate of $P'(x,z)$ at maximum β_x location as

$$\tan\phi = \sqrt{\frac{\beta_{x_0}\beta_{xm}}{\beta_{z_0}\beta_z}}\frac{z}{x} \quad (7)$$

The maximum value of ϕ i.e. ϕ_{xm} is obtained at location P' of electron loss for which $x = a$ *and* $z = \sqrt{\beta_z/\beta_{zm}}b$, so

$$\tan\phi_{xm} = \sqrt{\frac{\beta_{x_0}\beta_{xm}}{\beta_{z_0}\beta_z}}\sqrt{\frac{\beta_z}{\beta_{zm}}}\frac{b}{a} \Rightarrow \tan\phi_{xm} = \sqrt{\frac{\beta_{x_0}\beta_{xm}}{\beta_{z_0}\beta_{zm}}}\frac{b}{a} \Rightarrow \phi_{xm} = \tan^{-1}\left(\frac{pb}{a}\right) \quad \textit{where } p = \sqrt{\frac{\beta_{x_0}\beta_{xm}}{\beta_{z_0}\beta_{zm}}} \quad (8)$$

$$\text{From Fig. 1(a)}, \tan\phi_x = \frac{z}{x} \Rightarrow \tan\phi = p_1\tan\phi_x \quad \textit{where } p_1 = \sqrt{\frac{\beta_{x_0}\beta_{xm}}{\beta_{z_0}\beta_z}} \text{ (using eq. 7)} \quad (9)$$

Similarly from Fig. 1(b), at location of electron loss P', $x = \sqrt{\frac{\beta_x}{\beta_{xm}}}a$ and $z = b$, the maximum value of ϕ i.e.ϕ_{zm} is