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An analytical model for azimuthal thermoacoustic modes in an annular chamber fed by an annular plenum

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ABSTRACT

This study describes an analytical method for computing azimuthal modes due to flame/acoustics coupling in annular combustors. It is based on a quasi-one-dimensional zero-Mach-number formulation where N burners are connected to an upstream annular plenum and a downstream chamber. Flames are assumed to be compact and are modeled using identical flame transfer function for all burners, characterized by an amplitude and a phase shift. Manipulation of the corresponding acoustic equations leads to a simple methodology called ANR (annular network reduction). It makes it possible to retain only the useful information related to the azimuthal modes of the annular cavities. It yields a simple dispersion relation that can be solved numerically and makes it possible to construct coupling factors between the different cavities of the combustor. A fully analytical resolution can be performed in specific situations where coupling factors are small (weak coupling). A bifurcation appears at high coupling factors, leading to a frequency lock-in of the two annular cavities (strong coupling). This tool is applied to an academic case where four burners connect an annular plenum to a chamber. For this configuration, analytical results are compared with a full three-dimensional Helmholtz solver to validate the analytical model in both weak and strong coupling regimes. Results show that this simple analytical tool can predict modes in annular combustors and investigate strategies for controlling them.

1. Introduction

Describing the unstable acoustic modes that appear in annular gas turbine combustion chambers and finding methods to control them are the topic of multiple present research activities [1–9]. The complexity of these phenomena and the difficulty of performing simple laboratory-scale experiments explain why progress in this field has been slow for a long time since. Recently, the development of smaller annular chambers in laboratories has opened the path to investigating flow fields [10,11], ignition [12], flame response to acoustics [13], and azimuthal instabilities in these configurations [4,14,15]. At the same time, theoretical and numerical approaches have progressed in three directions: (1) full 3D LES of annular chambers has been developed [16,17], (2) 3D acoustic tools have been adapted to annular chambers [18–21], and (3) analytical approaches have been proposed to avoid the costs of 3D formulations and to investigate the stability and control of modes at low cost [5,22,23]. This last class of approaches is especially interesting for elucidating mechanisms (such as transverse forcing effect [23], symmetry breaking [5], and mode nature [24]) because they can provide explicit solutions for the frequency and the growth rate of all modes. The difficulty in these methods is in constructing a model that can be handled by simple analytical approaches while retaining most of the important physical phenomena and geometrical specificities of annular chambers.

One interesting issue in studies of instabilities in annular chambers is classifying them. For example, standing and turning modes [1,24] are both observed [1,5,17,25], but predicting which mode type will appear in practice and whether they can be studied and controlled with the same method remains difficult [23]. Similarly, most large-scale annular chambers exhibit multiple acoustic modes in the frequency range of interest (typically 10–30 acoustic modes can be identified in a large-scale industrial chamber between 0 and 300 Hz), and classifying them into categories is the first step in controlling them. These categories are typically “longitudinal vs azimuthal modes” or “modes involving only a part of the chamber (decoupled modes) vs modes involving the whole system (coupled modes)” [1,3,20,21]. Knowing that a given unstable mode is controlled only by a certain part of the combustor is an obvious asset for any control strategy. In the case of combustors including an annular plenum, burners, and an annular chamber,

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Nomenclature

$\alpha = z_{f,i}/L_i$	normalized abscissa for the flame location in the burners
$\beta = \frac{c^0 L_p}{c_u^0 L_c}$	tuning parameter
ϵ_p and ϵ_c	wavenumber perturbation in the plenum and chamber
$\Gamma_{i,k}$	kth coupling parameter of the ith sector
$\mathbb{F} = \frac{\rho^0 c^0}{\rho_u^0 c_u^0}(1 + n_i e^{j\omega\tau_i})$	flame parameter
ω	angular frequency
ρ_u^0 and ρ^0	mean density in the unburnt and burnt gases
τ_i	time delay of the FTF of the ith flame
$\tau_p^0 = \frac{2c_u^0}{pL_p}$ and $\tau_c^0 = \frac{2c^0}{pL_c}$	period of the unperturbed pth azimuthal mode of the plenum and chamber
θ	angle in the annular cavities
c_u^0 and c^0	mean sound speed in the unburnt and burnt gases
k_u and k	wavenumber in the unburnt and burnt gases
$L_c = \pi R_c$ and $L_p = \pi R_p$	half perimeter of the chamber or plenum
L_i	length of the ith burner
N	number of burners
n_i	interaction index of the FTF of the ith flame
p	order of the azimuthal mode
p'	pressure fluctuations
R_c and R_p	radii of the annular chamber or plenum
S_c, S_p, and S_i	cross sections of the chamber, plenum, and ith burner
u'	azimuthal velocity fluctuations (along x)
w'	axial velocity fluctuations (along z)
x	azimuthal abscissa in the chamber or plenum corresponding to $x_c = R_c\theta$ or $x_p = R_p\theta$
z	longitudinal abscissa in the burners
ANR	annular network reduction
ATACAMAC	analytical tool to analyze and control azimuthal modes in annular combustors
BC	burners + chamber configuration (Fig. 1 left)
BCp	burner + chamber mode of order p (the annular plenum is perfectly decoupled from the system)
FDCp	fully decoupled chamber mode of order p
FDPp	fully decoupled plenum mode of order p
FTF	flame transfer function
LES	large eddy simulation
PBC	plenum + burners + chamber configuration (Fig. 1 right)
PBp	plenum + burner mode of order p (the annular chamber is perfectly decoupled from the system)
SCp	strongly coupled mode of order p
WCCp	weakly coupled chamber mode of order p
WCPp	weakly coupled plenum mode of order p

such a classification is useful, for example, in understanding how azimuthal modes in the plenum and in the chamber (which have a different radius and sound speed and therefore different frequencies) can interact or live independently. For example, FEM simulations of a real industrial gas turbine [20] produce numerous complex modes that involve several cavities (plenum, burners, and chamber) at the same time. Unfortunately, determining whether certain parts of a chamber can be 'decoupled' from the rest of the chamber is a task for which there is no clear strategy. 'Decoupling' factors have been derived for longitudinal modes in academic burners where all modes are longitudinal [26]. Extending these approaches to annular systems requires first deriving analytical solutions that can isolate the effect of parameters on the modes structure. This is one of the objectives of this paper.

Noiray et al. [5] and Ghirardo and Juniper [23] have proposed analytical analysis of complex nonlinear mechanisms, but only in simple configurations where the combustor is modeled with an annular rig alone. Parmentier et al. [22] have derived an analytical method called ATACAMAC (for analytical tool to analyze and control azimuthal modes in annular combustors) for a more realistic configuration called BC (N burners + chamber) (Fig. 1 left). By describing acoustic wave propagation and flame action in a network of ducts representing the BC configuration and introducing a reduction method for the overall system corresponding to wave propagation in this network, they were able to predict the frequencies and growth rate of azimuthal and longitudinal modes, to identify their nature, and to predict their response to passive control methods such as symmetry breaking. Stow and Dowling [27] also investigated BC configurations with low-order models and focused on limit cycles by introducing more complex flame models (by adding nonlinearities and uniform spread of convection times).

However, BC geometries did not correspond exactly to real annular chambers where the N burners are connected not only downstream to the combustion chamber but also upstream to the plenum that feeds them. PBC configurations (plenum + N burners + chamber) (Fig. 1 right) have been proved [1,20] to correctly reproduce the behavior of complex industrial annular combustors (Fig. 2). Evesque and Polifke [8] studied PBC configurations using both FEM simulations and low-order models. Pankiewitz and Sattelmayer [21] also investigated this case using time-domain simulations with an axial mean flow. Both the linear and nonlinear flames regimes are studied by introducing saturations in the flame transfer functions. They pointed out that the time delay of the FTF plays a crucial role in predicting the frequency as well as the nature of azimuthal modes in such a configuration.

Thus, no full analytical resolution of the frequency and nature of azimuthal modes has been achieved in PBC configurations. The present paper extends the analytical methodology of Schuller et al. [26] for longitudinal tubes and of Parmentier et al. [22] for BC configurations to a PBC configuration with N burners in order to highlight key parameters involved in the coupling mechanisms.

In most network approaches to combustion instabilities, a very large matrix is built to describe the acoustics of the system [8,26,28]. Here, we introduce a significantly simpler methodology called ANR (annular network reduction) that makes it possible to reduce the size of the acoustic problem in an annular system to a simple 4-by-4 matrix containing all information of the combustor resonant modes. This method makes it possible to obtain explicit dispersion relations for PBC configurations and to exhibit the exact forms of the coupling parameters for azimuthal modes between the plenum and the burners, on one hand, and between the burners and the chamber, on the other hand.

The paper is organized as follows: Section 2 describes the principle of the ANR (annular network reduction) methodology and the submodels that account for active flames. The decomposition of the network into H-shaped connectors and azimuthal propagators makes it possible to build an explicit dispersion relation giving the frequency, growth rate, and structure of all modes. In Section 3, thermoacoustic regimes (from fully decoupled to strongly coupled) are defined, depending on the analytical coupling parameters conducted in Section 2. Finally, this analytical model is validated using the model annular chamber described in Section 4 with simplistic shapes to construct coupling factors and study azimuthal modes for a case where a plenum is connected to a chamber by four similar burners ($N = 4$). First the weakly coupled regime (Section 5) and then the strongly coupled regime (Section 6) are investigated.