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Conductance calculation of channels in laminar gas flow regime at an arbitrary pressure difference

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ABSTRACT

The conductance of channels formed by cylindrical walls and of rectangular channels in a wide range of pressures and pressure ratios was defined with the help of numerical solution of Navier–Stokes equations system. The mathematical model was verified. The results of the numerical solution were approximated in the form of equations which can be used in mathematical models of non-contact pumps and compressors.

1. Introduction

Pumping characteristics of non-contact pumps and compressors, for example, Roots, scroll, screw, claw pumps and compressors, to a large extent are influenced by backward leakage through channels between the rotors and between the rotors and shell walls. That is why the basis of mathematical model of such a machine is equations for leakages through narrow channels of different geometry. Geometry of channels may be very different.

In articles dedicated to studies of non-contact pumps in viscous flow regime [1–3] equations based on the formula for a long rectangular channel are most commonly used for conductance calculation of channels

$$G = \frac{L\delta^3}{12R_GT\eta}P\frac{\mathrm{d}P}{\mathrm{d}l}\left[1-\frac{192}{\pi^5}\frac{\delta}{L}\sum_{j=0}^{\infty}\frac{1}{(2j+1)^5}\cdot tan\delta\left(\frac{(2j+1)\pi}{2}\frac{L}{\delta}\right)\right], \quad (1)$$

where P_1 and P_2 are pressures at the inlet and the outlet of the channel, respectively; δ is the minimal clearance between the channel walls; L is the channel depth (the dimension in the direction perpendicular to the sheet of paper); l is the length of the channel in the direction of gas flow; η is the gas dynamic viscosity coefficient; R_G is the gas constant; T is the gas temperature.

Since clearances in non-contact machines are tenths or hundredths of a millimetre then $L >> \delta$ and Equation (1) is written in the following form

$$G = \frac{L\delta^3}{12R_GT\eta}\cdot\frac{P_1+P_2}{2l} \quad (2)$$

The main reason for not using Equations (1) and (2) for calculation of non-contact machines is the fact that they were obtained on the condition of very small pressure difference at the channel ends. But in non-contact vacuum pumps the pressure ratio is close to the critical pressure ratio. Such approach is quite justified for flank channels of non-contact pumps which amount to long rectangular channels but to use Equations (1) and (2) for profile channels with variable cross section along the channel length is unjustified.

In most studies on non-contact pumps in viscous flow regime, for example, in [1–4], influence of channel walls speed on conductance is neglected in view of small contribution in comparison with gas flow due to pressure difference.

Most channels of rotor non-contact pumps represent channels with variable cross section along gas flow direction and with the minimal clearance at a certain point (Fig.1). Formulae for Laval nozzle are often used for leakage calculation through such channels, for example [5].

But as it was shown in [6] conductance of such channels is determined mostly by the section of the channel near the minimal clearance. Therefore, the section giving the main resistance

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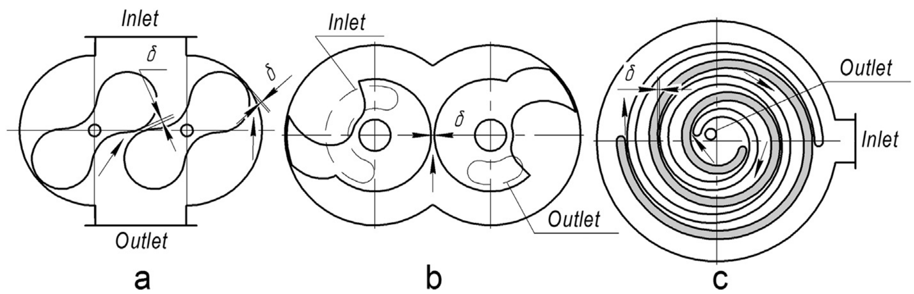

Fig. 1. Types of channels in non-contact vacuum pumps a – Roots pump; b – screw pump; c – scroll pump.

to the flow has small length, and channel walls with nearly any profile (differentiable curves) may be approximated there by convex and concave circle arcs with radii R_1 and R_2. Then, it is necessary to have formulae for channels formed by concave and convex cylindrical walls in order to calculate conductance of profile channels.

In [6] the simple relationship for conductance calculation of three types of channels formed by cylindrical walls (channels 1–3 in Fig.2) was analytically obtained in the following form

$$U = \frac{\sqrt{2}\delta^2 L\sqrt{\frac{\delta}{R_2} \pm \frac{\delta}{R_1}}P_{av}}{9\pi\eta} \qquad (3)$$

The sign of the curvature radius in (3) is chosen with regard to the curvature direction of the wall: the sign is" plus" for the channels 1and 3, the sign is “minus” for the channel 2.

But Equation (3) has a considerable restriction, i.e. it may be used only for a small pressure difference at the channel ends ($\tau = P_2/P_1 \to 1$).

Along with that, in non-contact pumps and compressors the pressure ratio at the ends of channels may vary in a very wide range, including values when local sound velocity takes place in the channel. Under such conditions even slight deviation of τ from unity results in deviation of relationship $U = f(P_{av})$ from linearity, and Equation (3) can't be used.

In this work laminar gas flow through channels of four types at an arbitrary pressure difference, including supercritical, is considered.

2. Mathematical model and assumptions

To define mass flow rate through the channels 1–4 in laminar flow regime the numerical solution of differential equations system (4)–(7), including the equation of motion, the continuity equation, the equation of state and the equation of energy, is used. The calculation is realized with the help of software ANSYS (Fluent) (ANSYS Academic Research CFD license file for Kazan State Technological University, Customer Number: 607451 created 01.01.2011).

$$\frac{\partial(\rho)}{\partial t} + div(\rho W) = 0, \qquad (5)$$

$$\frac{\partial}{\partial t}(\rho E) + div(W(\rho E + p)) = div(\lambda \mathrm{grad}(T)), \qquad (6)$$

$$\rho = p/RT, \qquad (7)$$

where ρ is the density; w_x, w_Y, w_Z are velocity components in x, y, z directions, respectively; X, Y, Z are volume forces; E is the energy; λ is the heat conductivity coefficient; R is the universal gas constant; p is the pressure.

As it was mentioned above, for all considered channels of non-contact vacuum pumps the width L of the channel is many times more than the clearance, and numerical solution shows that for $L/\delta > 50$ the conductance changes not more than 1%. That is why two-dimensional model for steady-state flow is used for the solution.

For numerical solution of the equations system, the method of control volume is used.

The boundary conditions were defined as follows: sticking on the channel walls and uniform cross section pressure profiles at the inlet and the outlet channel ends were specified. The deviation of the actual cross section pressure profile and the uniform cross section pressure profile is small because the inlet and the outlet cross sections are considerably remote from the minimal clearance position (channels 1–3) and, thereafter, from the maximal resistance and pressure gradients position. The channel length was taken under the condition of more than 20 times widening in relation to the minimal clearance dimension. Air was taken as a working gas and the dynamic viscosity coefficient was calculated with the help of Sutherland formula for different temperatures. Heat exchange between the gas and the channel walls is lacking.

For a rectangular channel the effect of pressure profile cross section form at the inlet cross section is small as we studied channels with $l/\delta > 20$, therefore, the effect of the initial section with unsteady flow is negligible.

To define convective terms the first-order upwind was used. For calculation of pressure–velocity coupling algorithm Simple was used (Semi-Implicit Method for Pressure-Linked Equations).

$$\begin{aligned}
\rho\left(\frac{\partial w_X}{\partial t} + w_X\frac{\partial w_X}{\partial x} + w_Y\frac{\partial w_Y}{\partial y} + w_Z\frac{\partial w_Z}{\partial z}\right) &= X - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left[\mu\left(2\frac{\partial w_X}{\partial x} - \frac{2}{3}div(W)\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(2\frac{\partial w_X}{\partial y} + \frac{\partial w_Y}{\partial x}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(2\frac{\partial w_Z}{\partial x} + \frac{\partial w_X}{\partial z}\right)\right],\\
\rho\left(\frac{\partial w_Y}{\partial t} + w_X\frac{\partial w_Y}{\partial x} + w_Y\frac{\partial w_Y}{\partial y} + w_Z\frac{\partial w_Y}{\partial z}\right) &= Y - \frac{\partial p}{\partial y} + \frac{\partial}{\partial y}\left[\mu\left(2\frac{\partial w_Y}{\partial y} - \frac{2}{3}div(W)\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(2\frac{\partial w_Y}{\partial z} + \frac{\partial w_Z}{\partial y}\right)\right] + \frac{\partial}{\partial x}\left[\mu\left(2\frac{\partial w_X}{\partial y} + \frac{\partial w_Y}{\partial x}\right)\right],\\
\rho\left(\frac{\partial w_Z}{\partial t} + w_X\frac{\partial w_Z}{\partial x} + w_Y\frac{\partial w_Z}{\partial y} + w_Z\frac{\partial w_Z}{\partial z}\right) &= Z - \frac{\partial p}{\partial z} + \frac{\partial}{\partial z}\left[\mu\left(2\frac{\partial w_Z}{\partial z} - \frac{2}{3}div(W)\right)\right] + \frac{\partial}{\partial x}\left[\mu\left(2\frac{\partial w_Z}{\partial x} + \frac{\partial w_X}{\partial z}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(2\frac{\partial w_Y}{\partial z} + \frac{\partial w_Z}{\partial y}\right)\right],
\end{aligned} \qquad (4)$$