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Dispersion relation of a plasmonic waveguide on substrate

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ABSTRACT

Dispersion relation of the surface plasmon polariton (SPP) propagating along a subwavelength gold waveguide deposited on a substrate is studied theoretically. The eigenvalues of a 1D waveguide are calculated using the finite element method combined with the method of moments in the visible-NIR regime. The dispersion curve is obtained directly by calculating the complex wavenumber versus real frequency, rather than the complex frequency versus real wavenumber. This method particularly benefits the problem with frequency-dependent material property. The shape effect (square or rectangle) of a gold nanowire and the refractive index of substrate on the propagation length, phase velocity and group velocity are investigated. Moreover, the mode profile is analyzed to evaluate the local confinement of SPP's evanescent field. In addition, the analytical solution of a circular nanowire is provided for comparison. Our results show that a plasmonic waveguide is a lowpass filter with a cutoff frequency, which decreases as the substrate's refractive index increases.

1. Introduction

Surface plasmon polaritons (SPPs) along metallic nanowires have become one of the interesting and useful research topics of nanophotonics [1–4]. The common wavelengths used by a conventional optical communication system are in the near infrared (NIR) regime (e.g. 850 nm, 1310 nm, and 1550 nm), where the SPP spectrum of a subwavelength plasmonic waveguide is also located. Therefore, plasmonic devices on a photonic integrated circuit (PIC) can be easily integrated with the conventional optical system. Recently, a variety of plasmonic waveguides have been proposed and studied [2–18], e.g. V-groove in metal substrate [7,8]. A useful and fundamental plasmonic waveguide for PIC is a metallic nanowire with uniform cross section deposited on a dielectric substrate [1,2]. To minimize the cross-section size of a plasmonic waveguide can raise the density of PIC, but also reduce the propagation length of SPP. In addition, the local confinement of SPP's evanescent field in the lateral side of a plasmonic waveguide is also concerned to avoid the crosstalk of two adjacent waveguides [19]. Therefore, it is tradeoff to downsize the plasmonic waveguide for implementing a high-density PIC.

In this paper, the dispersion relation (propagation constant versus wavelength) of SPP along a gold waveguide deposited on a dielectric substrate in the visible-light to NIR regime is studied numerically, where the cross section of the waveguide is a rectangle of subwavelength size. A new technique combining the flexibility of the finite element method (FEM) and the efficiency of the method of moments (MoM) [20] is employed to compute the eigenvalues (i.e. propagation constants) of plasmonic waveguides for a given wavelength (or frequency). The advantage of this method for the dispersion curve is the direct calculation of the complex wavenumber for a given real frequency. In contrast, the commercial 3D fullwave solvers (e.g. ANSOFT HFSS) find the dispersion relationship by calculating the complex frequency for a given real wavenumber. Since the permittivity of gold is frequent-dependent, it is more convenient to obtain the dispersion curve by directly calculating the complex wavenumber versus real frequency than the complex frequency versus real wavenumber. The hybrid MoM/FEM technique in Ref. [20] was originally proposed to solve 3D periodic structures. However, the same formulation can also be used to study the dispersion relation of a 2D waveguide structure. The effects of the shape of gold waveguide (square or rectangle) and the

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refractive index of the substrate on the dispersion properties (propagation length, phase velocity and group velocity) are investigated. In addition, the mode profile is also analyzed to examine the local confinement of the evanescent field of SPP in the surrounding medium.

2. Theory and method

Throughout the paper, the time harmonic factor for Maxwell's equations is $\exp(j\omega t)$, where ω is the angular frequency and $j = \sqrt{-1}$. The frequency-dependent permittivity of gold is referred to Ref. [21]. The permittivity of a lossy material (e.g. gold) is expressed as $\varepsilon{=}\varepsilon' - j\varepsilon''$, where the imaginary part is positive; $\varepsilon'' > 0$. The imaginary part of the permittivity of gold causes the ohmic loss of a plasmonic waveguide. Fig. 1 shows the configuration of a rectangular plasmonic waveguide deposited on a dielectric substrate, where w and h denote the width and height, respectively. The SPP propagates along y-axis, the long axis of the 2D plasmonic waveguide with a uniform cross section. The wavenumber (or called propagation constant) of SPP is expressed as $k = k_r - jk_i$ for a given frequency, where the imaginary part is related to the attenuation of SPP, $k_i > 0$. The propagation length is defined as $l = 1/k_i$. The phase velocity of the SPP is given by $v_p = \omega/k_r$, and the group velocity $v_g = \partial\omega/\partial k_r$.

To find the dispersion relationship of a conventional waveguide, the commercial 3D fullwave solvers (e.g. ANSOFT HFSS) are usually used. Normally the algorithm of commercial package requires a user to specify a real wavenumber first, and then finds the corresponding frequency. Since the material of a plasmonic waveguide (e.g. gold) is lossy, the resulting frequency is a complex value. Subsequently, an approximate formula can be used to inversely derive the equivalent complex wavenumber versus a real frequency. This procedure is cumbersome, and correct only for low loss cases. Since the material property of gold is frequency-dependent, this method causes a difficulty of calculating the dispersion curve. Therefore, we developed a program based on the hybrid MoM/FEM technique [20] to study the plasmonic waveguide. Unlike the disadvantages of the commercial software (HFSS), our method directly calculates the complex wavenumber as a function of a real frequency. It is more convenient to obtain the dispersion curve by the wavenumber versus frequency than the frequency versus wavenumber, particularly when the material property is frequency-dependent. In the hybrid MoM/FEM technique, the real frequency ω is specified first. Then, a section of the waveguide with length L is modeled and simulated. The cosine of the resulting complex phase delay $\varphi = (k_r - jk_i)L$ is computed by solving the eigenvalues of a system of linear equations. Thus, the dispersion curve, the complex wavenumber as a function of real frequency, can be obtained. However, a judicious choice of L is required. A larger L takes longer to simulate, while a smaller L gives less accurate result. Since $\cos\varphi$ is computed first, the correct branch of the inverse cosine function should be carefully chosen when computing φ. In our experience, a 0.2 times of the average wavelength, which is computed by averaging the dielectric constants of the air, the substrate and gold, gives satisfactory results.

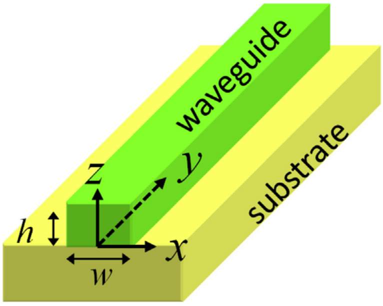

Fig. 1. Configuration of a rectangular plasmonic waveguide ($w \times h$) deposited on a dielectric substrate.

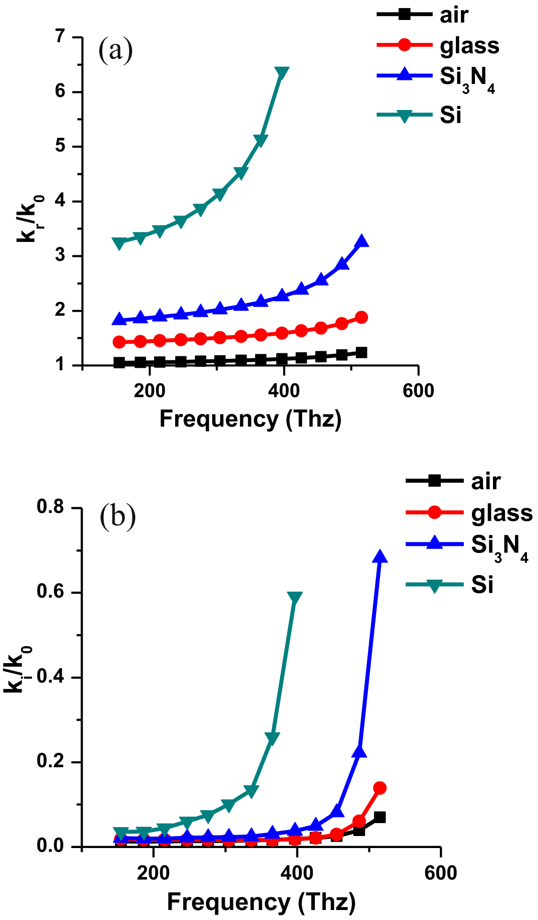

Fig. 2(b)

Fig. 2. Normalized (a) real parts and (b) imaginary parts of wavenumbers of SPP along a square gold waveguide (200 nm × 200 nm) on different substrates (air, glass, Si_3N_4, Si) versus frequencies.

3. Results and discussion

The complex wavenumber of SPP along a 2D rectangular gold waveguide ($w \times h$) on a dielectric substrate is calculated numerically for a given frequency using the hybrid MoM/FEM technique. Throughout this paper, the upper surrounding medium is air. The dispersion relations of a rectangular gold waveguide on a substrate with different refractive indexes are analyzed: $n = 1.5$ for glass, $n = 2$ for silicon nitride, and $n = 3.42$ for silicon (Si). Fig. 2(a) and (b) show the normalized real and imaginary parts of wavenumber (k/k_0) of SPP as propagating along a square gold waveguide (200 nm × 200 nm) on different substrates versus frequencies. Here k_0 is ω/c, where c is the light speed in vacuum. The results of a rectangular gold waveguide (200 nm × 100 nm) are plotted in Fig. 3. Both cases exhibit that a plasmonic waveguide is a lowpass filter with a cutoff frequency, indicated by the abrupt increase in the normalized real and imaginary parts of wavenumber. This is to say that the attenuation of SPP along a plasmonic waveguide becomes significantly serious when the frequency is higher than the cutoff frequency. Moreover, the higher the refractive index of