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Rarefied gas flow through a zigzag channel

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ABSTRACT

A rarefied gas flow through a channel of zigzag shape is calculated over the wide range of the gas rarefaction and for several values of the aspect ratio applying the linearized kinetic equation. In the hydrodynamic flow regime, the kinetic solution is compared with that obtained from the Stokes equation. An approach to calculate a flow rate through a chain composed from an arbitrary number of zigzags is proposed. It was found that in some situations, the flow rate through a zigzag channel is higher than that trough a straight channel of the same length.

1. Introduction

In modeling of rarefied gas flows through long pipes, it is quite usual to consider a straight channel or tube, see e.g. Refs. [1–6]. Such results are used to model gas flow in microfluidics [7], vacuum systems [8], in crevices [9] etc. In practice, one deals with pipes having bends and/or elbows, i.e. the pipes are not straight. Moreover, a crevice appeared as a result of cracks of vacuum chambers is not straight and it has a complicated shape, for instance, the shape of saw teeth. As a result, the flow rate of gas through non-straight channel can be significantly different from that estimated for a straight one.

Some results on gas flow through a single elbow or bend are reported in Refs. [10–14]. The flow in the single 90° corner was simulated in Ref. [10] using the DSMC method and no flow separation was found even for the high flow rate. The authors of Ref. [11] implemented the compressible Navier–Stokes equation to the numerical simulations of the slip flow through microchannel with two 90° bends. They reported that in the considered twisted geometry the mass flow rate is reduced by ~160% in comparison with that for a straight channel with the same overall dimensions. The gas flow through channels with a right-angled bend has been numerically analyzed in Ref. [12] using the incompressible Navier–Stokes equations with the velocity slip boundary conditions to study the effect of the fillet radius on the flow characteristics. The flow separation was found for the fillet radius equal to zero. The Lattice Boltzmann method was implemented in Ref. [13] to study the gas flow in a single 90° microchannel. The mass flow rate calculated for the Knudsen number varying in the range 0.1–0.5 is slightly greater than that of the straight channel of the same length when the same pressure difference was applied. The mass flow rates and the pressures losses are measured in the work [14] for three bend configurations: miter, curved and double-turn. The mass flow rate in all devices is found smaller than that of a straight microchannel, namely the lowest flow rate, about 70% of the straight channel rate, was measured in the miter-bend. The detailed pressure measurements indicate the flow separation in microchannel with miter-bend.

However, the situation when a channel has many bends, i.e. it consists of a chain of bends, has not been studied yet. In such a situation, a two-dimensional periodic gas flows should be considered. Some examples of other shapes of periodic flows of rarefied gases were considered in Refs. [15–17].

The aim of the present paper is to calculate a rarefied gas flow through a channel of zigzag shape over a wide range of the gas rarefaction and for several values of the channel aspect ratio.

2. Statement of the problem

Consider a channel of zigzag shape connecting two chambers as is shown in Fig. 1. A pressure in the left chamber is maintained equal to p_1 and that in the right chamber is equal to p_2. For the sake of certainty, we assume $p_1 > p_2$. The number of the elbows N is assumed to be so large that the channel is represented as a chain of

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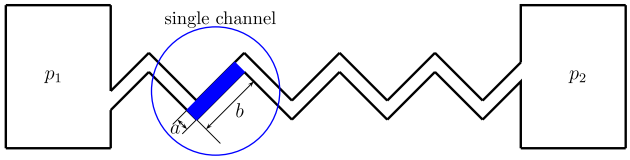

Fig. 1. Scheme of zigzag channel.

periodic structures. The element of this chain can be chosen by several ways, e.g. an elbow or straight short channel. In order to simplify the further calculations, the straight channel emphasized in Fig. 1 will be considered as a single element of the chain. A scheme of this straight single channel is depicted in Fig. 2, where a is the channel height and b is its length. The channel width in the z-direction is assumed to be sufficiently large so that the gas flow can be considered as two-dimensional.

Thus, the solution of the problem can be divided in two stages. First, a gas flow through a single channel is calculated. In the second stage, the methodology to calculate a flow rate through a long tube or channel at arbitrary pressure drop elaborated in Refs. [1,2,18–20] is used to calculate the flow rate through a channel composed from many channels connected in the zigzag manner.

3. Flow through a single channel

3.1. Input equation

Consider a two-dimensional flow through a straight channel shown in Fig. 2. The gas inflows into the channel through the surface fixed at $y' = a$ and stretched over the interval $0 \le x' \le a$. The outlet of the gas is fixed at $x' = b$ with y' varying as $0 \le y' \le a$. The pressure drop between the inlet and outlet of the single channel is denoted as Δp, while p_0 denotes the average pressure in the channel. Since the number of the single channels N is large, i.e. the total length of the zigzag channel equal to Nb is large, the pressure drop Δp across each single channel can be assumed to be small compared to its average value p_0, i.e.

$$\Delta p \ll p_0, \tag{1}$$

even if the total pressure difference $(p_1 - p_2)$ is large.

The results will be presented in terms of the reduced flow rate G_P related to the mass flow rate per length unit in the z-direction $\dot{M}$ as

$$\dot{M} = G_P \frac{a^2 \Delta p}{b v_m}. \tag{2}$$

The coefficient G_P is determined by the aspect ratio

$$B = b/a \tag{3}$$

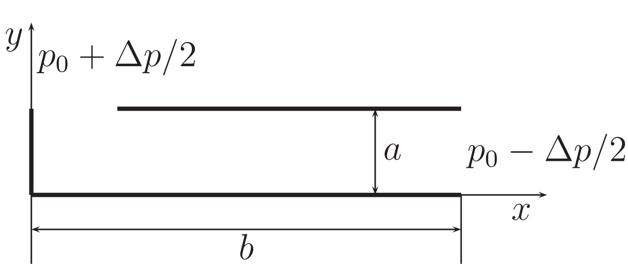

Fig. 2. Flow scheme and coordinates for single channel.

and by the local rarefaction parameter defined as

$$\delta = \frac{a p_0}{\mu v_m}, \quad v_m = \left(\frac{2kT_0}{m}\right)^{1/2}, \tag{4}$$

where μ is the gas viscosity and v_m is the most probable molecular speed, T_0 is the gas temperature in equilibrium, k is the Boltzmann constant and m is the molecular mass of the gas. The quantity $\ell = \mu v_m / p_0$ represents the equivalent free path so that the rarefaction parameter is inversely proportional to the Knudsen number.

Since we are going to consider the whole range of the rarefaction parameter, the problem must be solved on the level of the velocity distribution function $f(\boldsymbol{r}', \boldsymbol{v})$, where $\boldsymbol{r}'$ is the two-dimensional position vector and $\boldsymbol{v}$ is the molecular velocity. The number density n, bulk velocity $\boldsymbol{u}'$ and temperature T are calculated via the distribution function $f(\boldsymbol{r}', \boldsymbol{v})$ as

$$n = \int f \mathrm{d}\boldsymbol{v}, \quad \boldsymbol{u}' = \frac{1}{n}\int \boldsymbol{v} f \mathrm{d}\boldsymbol{v}, \quad T = \frac{m}{3nk}\int V^2 f \mathrm{d}\boldsymbol{v}, \tag{5}$$

where $\boldsymbol{V} = \boldsymbol{v} - \boldsymbol{u}'$.

The distribution function obeys the kinetic Boltzmann equation [1,21,22]. To reduce the computational effort, the Bhatnagar–Gross–Krook (BGK) model [23] is used here. The further derivations will be done in the following dimensionless quantities

$$x = \frac{x'}{a}, \quad y = \frac{y'}{a}, \quad \boldsymbol{c} = \frac{\boldsymbol{v}}{v_m}. \tag{6}$$

Since the pressure drop is small (1), the distribution function can be linearized as

$$f(\boldsymbol{r}, \boldsymbol{c}) = \frac{n_0}{(\sqrt{\pi} v_m)^3} \mathrm{e}^{-c^2}[1 + h(\boldsymbol{r}, \boldsymbol{c})\xi], \quad \xi = \frac{1}{B}\frac{\Delta p}{p_0}, \tag{7}$$

where n_0 is the equilibrium number density. Thus, the average pressure gradient ξ is used as the small parameter of the linearization. Substituting this representation into (5), we obtain the linearized form of the moments

$$n(x,y) = n_0[1 + \nu(x,y)\xi], \tag{8}$$

$$\boldsymbol{u}'(x,y) = v_m \boldsymbol{u}(x,y)\xi, \tag{9}$$

$$T(x,y) = T_0[1 + \tau(x,y)\xi], \tag{10}$$

where

$$\begin{bmatrix} \nu \\ \boldsymbol{u} \\ \tau \end{bmatrix} = \frac{1}{\pi^{3/2}} \int \begin{bmatrix} 1 \\ \boldsymbol{c} \\ \frac{2}{3}c^2 - 1 \end{bmatrix} \mathrm{e}^{-c^2} h(x,y,\boldsymbol{c}) \mathrm{d}\boldsymbol{c}. \tag{11}$$

Combining Eqs. (2) and (11), the reduced flow rate is obtained as

$$G_P = 2\int_0^1 u_x(x,y)\mathrm{d}y, \quad 1 \le x \le B. \tag{12}$$

As has been noted above, the coefficient G_P is a function of the aspect ratio B and rarefaction parameter δ, i.e. $G_P = G_P(B,\delta)$. In the case of infinite channel, i.e. $B \to \infty$, the quantity G_P tends to that defined in Ref. [1] for an infinite channel.

The linearized BGK model in our notations reads

$$c_x \frac{\partial h}{\partial x} + c_y \frac{\partial h}{\partial y} = \delta\left[\nu + 2\boldsymbol{c}\cdot\boldsymbol{u} + \tau\left(c^2 - \frac{3}{2}\right) - h\right]. \tag{13}$$